GOODYEAR AEROSPACE
CORPORATION

AKRON 15, OHIO

GOODYEAR AEROSPACE CORPORATION

ADVANCED PASSIVE COMMUNICATIONS
LENTICULAR SATELLITE STUDIES

Contract NAS 1-3114
Amendment No. 6

Summary Report - Phase III

December 1964

GER-11891

Copy No. ____

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218-52 (6-63)
REF: ENGINEERING PROCEDURE S-017

National Aeronautics and Space Administration

Langley Research Center

Langley Station, Hampton, Virginia

TABLE OF CONTENTS

E-ID-15(7-64)(77-10)
REF: ENGINEERING PROCEDURE S-017

INTRODUCTION AND SUMMARY

This report gives the status of the subject contractual effort for Phase III covering the period of November 1964 to satisfy the report requirements of the contract.

A technical review meeting was held at NASA Headquarters at the request of NASA-LRC personnel where Goodyear Aerospace (GAC) and Westinghouse personnel summarized the results of the lenticular satellite studies conducted under contract to date.

Cognizant NASA-Headquarters, Goddard and Langley personnel were present to review technical progress in the lenticular satellite area and utilize this information for passive communications satellite systems planning. Copies of technical memoranda presented at the meeting and supplementary information are included in this report as Appendices for information only. Further information on program goals can be obtained in Reference 1. Reference 2 concerns Phase I of this program and covers the first two coordination meetings. Reference 3 concerns Phase II of this program and covers the third, fourth and fifth coordination meetings.

Appendix A is a copy of the flip charts used by GAC at the sixth coordination meeting at NASA-Headquarters to review the lenticular satellite work done under contract with NASA-LRC since July 1963. Page A-3 gives a good history of this work along with listing pertinent documents and milestone dates for further information. These charts provide summary information and the reader is directed to use the referenced documents for technical details.

Appendix B is a study of the lenticular satellite weight and provides evaluation of the areas of potential weight reduction. A summary of this information was presented in Appendix A.

Appendix C considers a tumbling satellite from a structural viewpoint. Pitch and roll axis tumbling were investigated to determine representative forces and moments acting on the tetrapod apex. The effects of these forces and moments on the tetrapod booms were also presented. A summary of this information was presented in Appendix A.

Appendix D is a study of the canister separation velocity for representative symmetric and asymmetric satellite configurations. Appendix E presents the moments of inertia of the baseline symmetrical satellite about its principal axes during deployment while Appendix F presents similar data for the baseline asymmetric configuration.

Copies of Appendix A were given to everyone at the December 15th meeting while advance copies of Appendices B and C were given to cognizant NASA-LRC personnel. Appendices D thru F are presented for information.

Revisions to the analyses are made from time to time as more information becomes available and new ideas are generated. The final report on the program will summarize the overall technical achievements and recommend future effort.

REFERENCES

1. GAP-2680 - Advanced Passive Communications Lenticular Satellite Studies May 1964

2. GER-11789 - Advanced Passive Communication Lenticular Satellite Studies October 25, 1964 Contract No. NAS 1-3114, Amendment 6, Phase I

3. GER-11816 - Advanced Passive Communications Lenticular Satellite Studies November 1964 Contract No, NAS 1-3114, Amendment 6, Phase II

LENTICULAR SATELLITE PROGRAM

CONTRACT NAS 1-3114

AMENDMENT 6

COORDINATION MEETING

NUMBER 6

NASA HEADQUARTERS

15 DECEMBER 1964

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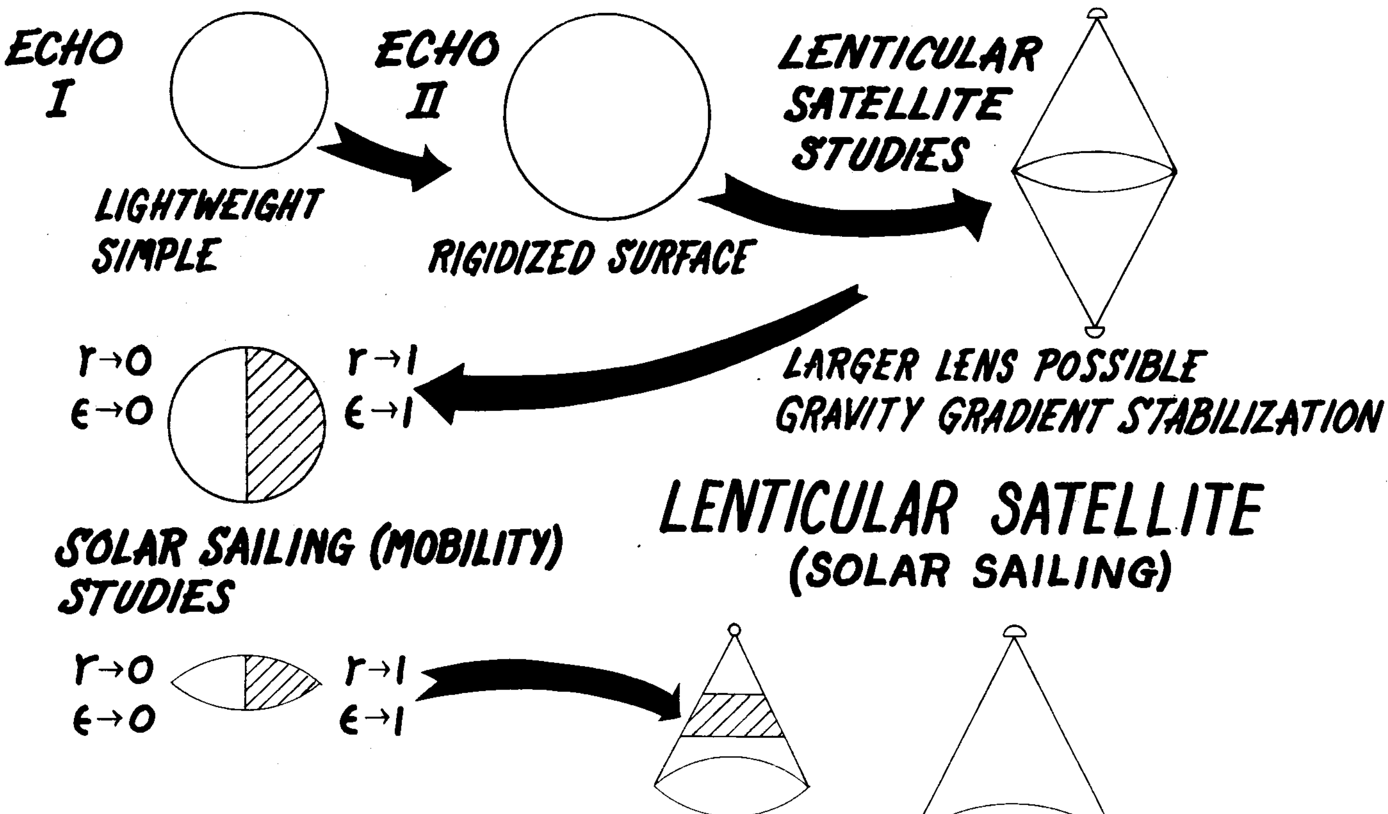
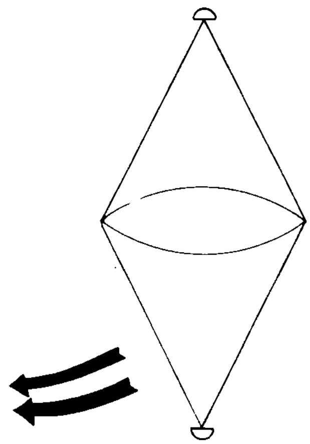
GOODYEAR

PASSIVE COMMUNICATION SATELLITE EVOLUTION





COMMUNICATION SYSTEM STUDIES

RF TESTING

MATERIALS IMPROVEMENT

SATELLITE DESIGN AND MOBILITY STUDIES

FLIGHT TEST PROGRAM

OPERATIONAL SYSTEM

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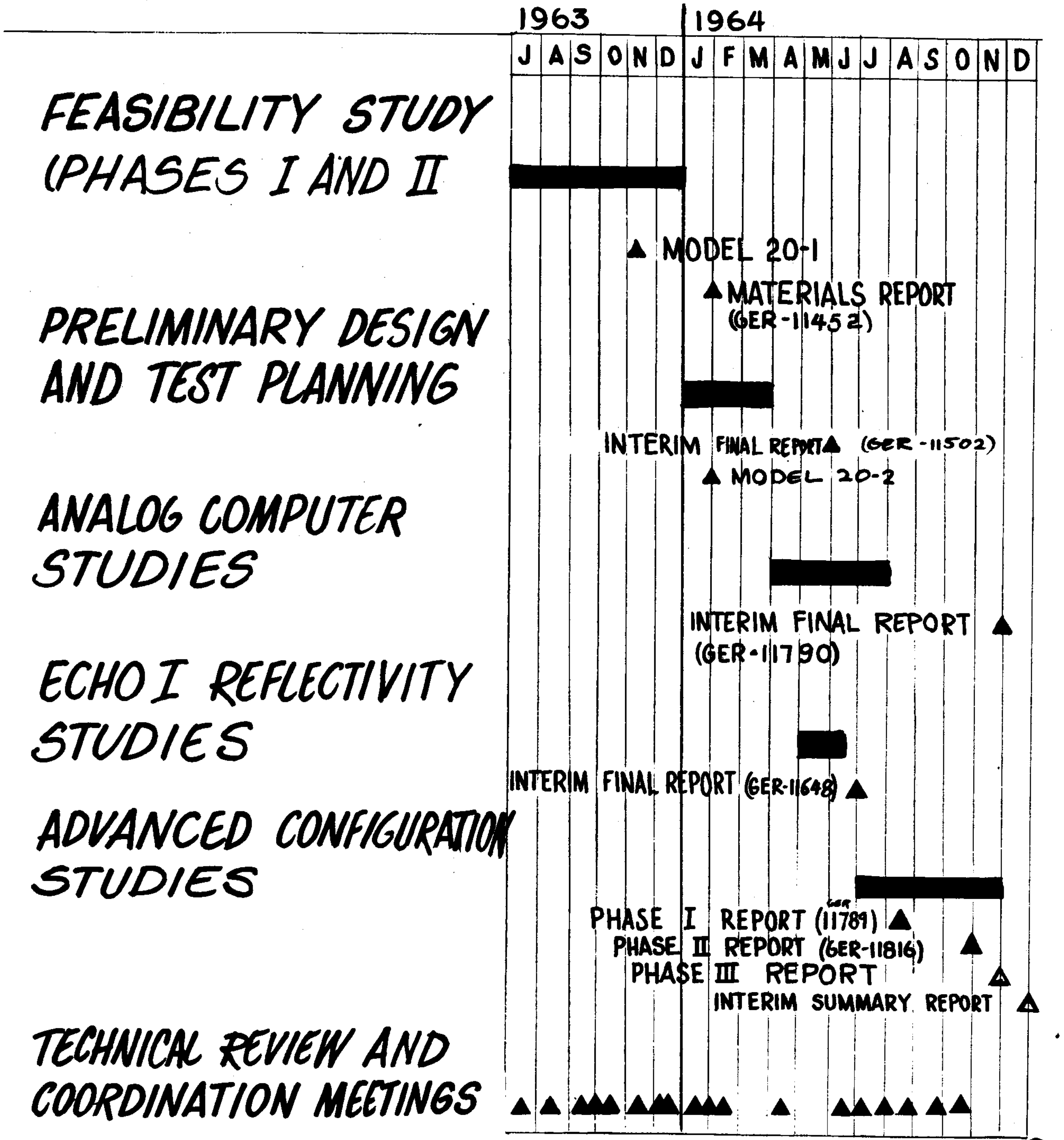
LENTICULAR SATELLITE DEVELOPMENT PROGRAM NAS 1-3114



DESIGN DEFINITION ALTERNATIVES

CONFIGURATION
- ASYMMETRICAL VS SYMMETRICAL
- SAIL VS OPAQUE LENS
- MATERIALS
- STRUCTURE

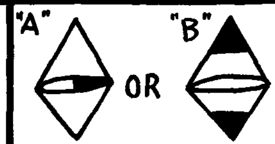
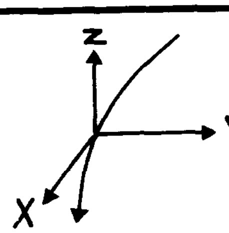
STABILIZATION
- CAPTURE →
  - SPRING-MASS
  - ARTICULATED BOOM (S)
  - HYSTERESIS (STRUCTURAL, MAGNETIC)

YAW CONTROL
- VENETIAN BLIND (PHILLIPS' CONCEPT)
- CONTROLLED YAW ANGLE →
  - TWO ANGLES
  - MULTIPLE ANGLES
  - COIL
  - REACTION
  - INERTIA DISTRIBUTION
    - CANISTER DRIVE
    - RIM DRIVE

ELECTRONICS
- ON-BOARD COMPUTER VS ON-GROUND COMPUTER
- COMMAND RECEIVER
- INSTRUMENTATION

POWER SUPPLY
- SOLAR CELL/BATTERY
- RADIOISOTOPE

DESIGN GUIDELINES AMENDMENT 6

	ORIGINALLY	PRESENT STATUS
SAIL CONFIGURATION STUDY		
MOBILITY	100 DEG/MO	100 DEG/MO
ORBIT ALTITUDE	2000 N MI	2000 N MI
ORBITAL LIFE	5 YR	5 YR
ORBIT INCLINATION	0 TO 90 DEG	60 TO 65 DEG
YAW CONTROL POSITIONS	OSCILLATING (PHILLIP'S CONCEPT)	3 OR 5
YAW CONTROL TOLERANCE	—	± 30 DEG OPAQUE LENS ± 10 DEG FLAT SAIL
VERTICAL POINTING ERROR	3 DEG PITCH; 3 DEG ROLL	± 3 DEG
YAW CONTROL SETTLING TIME	—	~ 1 DAY
CONFIGURATION	(PER GER-11502)	(PER GER-11502)
 $I_Z' \approx 10^5$ SLUG-FT^2 $I_Y \approx (10^6+\Delta)$ SLUG-FT^2 $I_X = 10^6$ SLUG FT^2 $\Delta = 1$ TO 5% OF I_X	LENS DIAM -267.6 FT LENS RADIUS -200 FT	LENS DIAM -267.6 FT LENS RADIUS -200 FT

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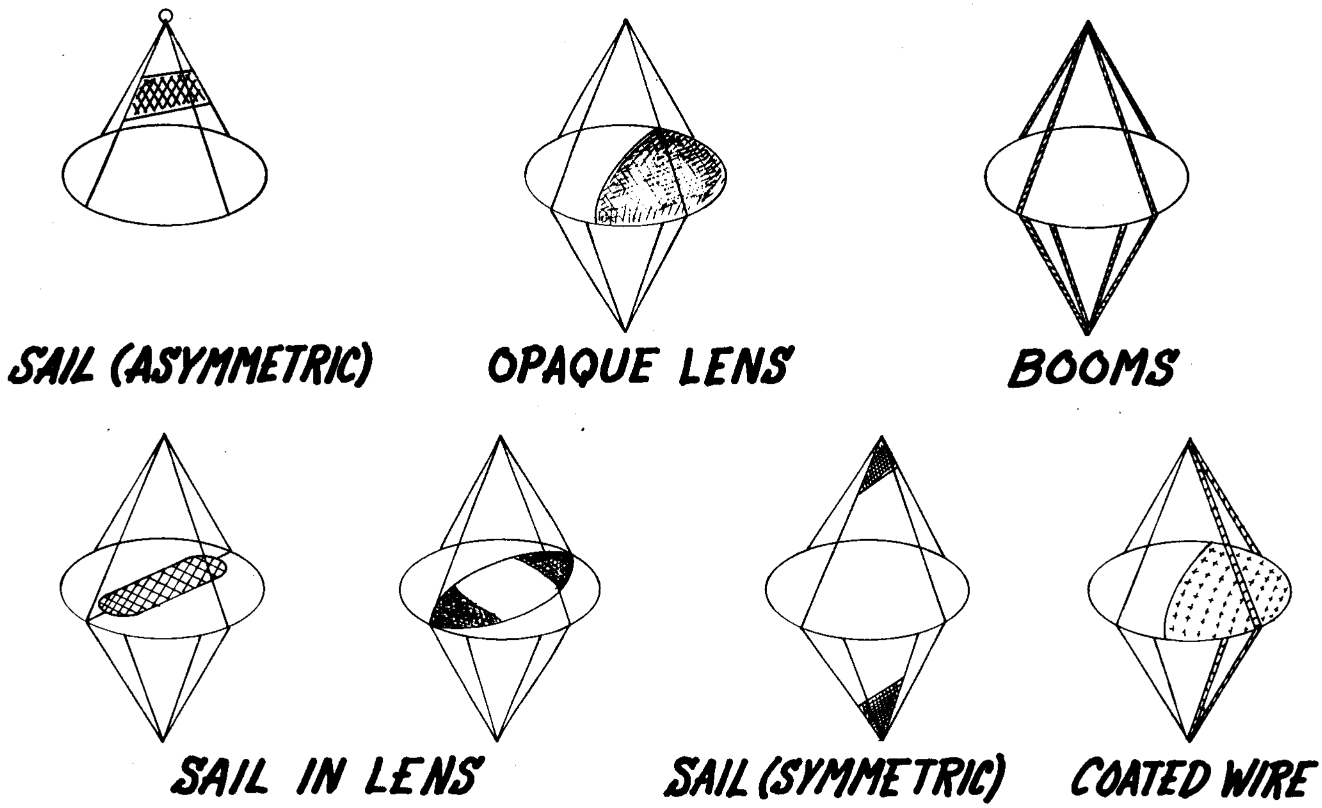
SAIL CONFIGURATION

ALTERNATIVES AND REQUIREMENTS



MOBILITY - 100 DEG/MONTH

OPTICAL CHARACTERISTICS - DIFFUSE

ENERGY INPUTS - SOLAR RADIATION, ALBEDO, RERADIATION

UPSETTING TORQUES - MINIMUM OR BALANCED

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-5-

SAIL CHARACTERISTICS

MINIMUM REQUIREMENTS	RECOMMENDED SOLUTIONS

FLAT SAIL

$r_1 = 0.9$ | $r_2 = 0.1$

$\varepsilon_1 = 0.8$ | $\varepsilon_2 = 0.2$

← $r_1 = 0.8$ (DIFFUSE) $\varepsilon_1 = 0.8$ | $r_2 = 0.28$, $\varepsilon = 0.05$ } GAC TEST DATA

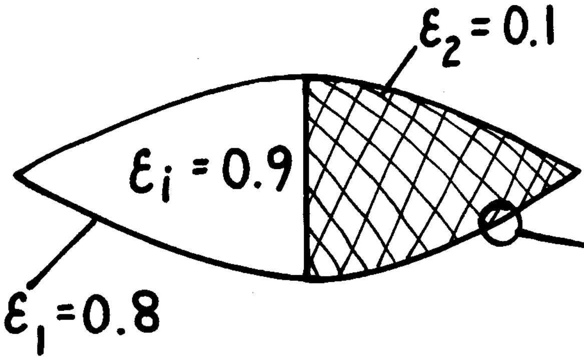
DARK MIRROR-VAPOR DEPOSITS (Al-SiO-Al-SiO)

1/4-MIL MYLAR

VAPOR DEPOSIT - METAL OXIDE ON ALUMINUM

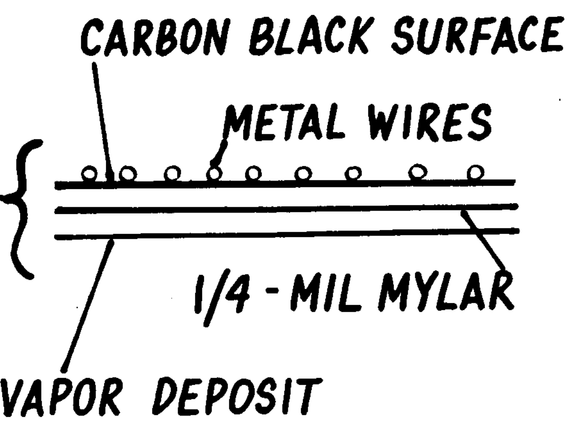
WEIGHT $= 2.43\,(10^{-3})$ LB/FT^2

OPAQUE LENS



$\alpha = 0.35$

$r = 0.65$



WEIGHT $\approx 3.47\,(10^{-3})$ LB/FT^2

STATUS OF RELATED MATERIAL DEVELOPMENT AND TEST

THERMAL COATING STUDIES

PIGMENTED SURFACE COATINGS

ROTOFLEX TESTS - CHECK ADHESION TO MYLAR (-25°C, 23°C, 100°C)

OPTICAL CHARACTERISTICS - α_S, ϵ (ROOM TEMP)

EFFECT OF UV - 1000 EQUIVALENT SUN HOUR EXPOSURE, VACUUM - 10^{-6} MM Hg

SOLAR SAIL MATERIALS

DARK MIRROR SURFACE TESTS (RT)

UV EXPOSURE IN VACUUM - 1000 EQUIVALENT SPACE HOURS (10^{-6} MM HG)

HIGH r, HIGH E SURFACE TESTS (RT)

EFFECT OF UV ON α_S AND ϵ

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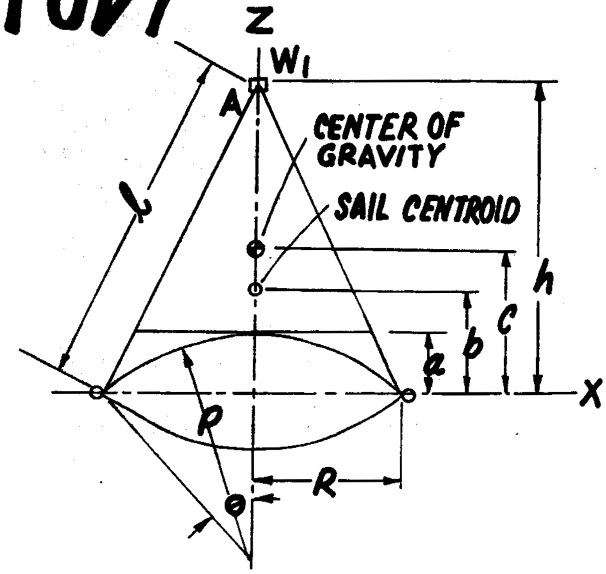
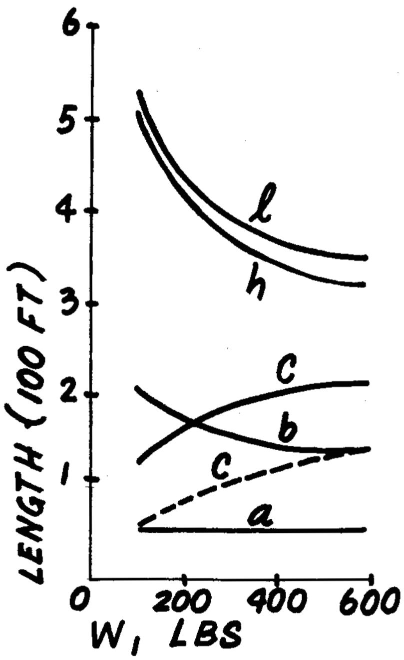
ASYMMETRICAL CONFIGURATION STUDY

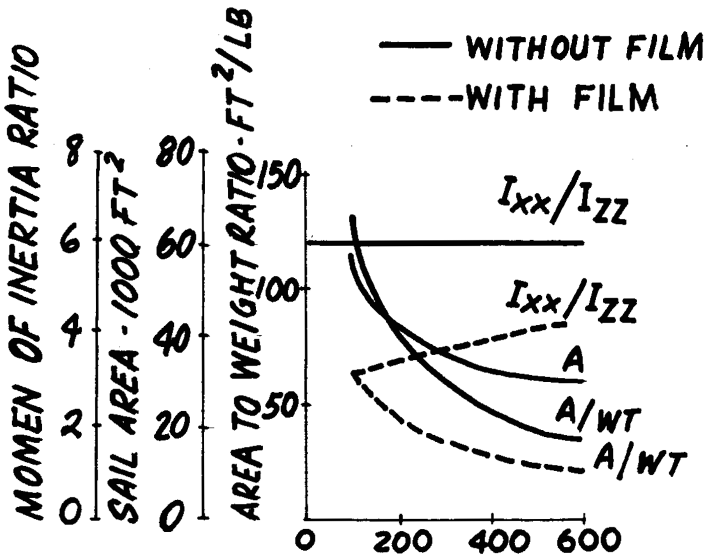


LENS, RIM, TORUS DATA

	WITHOUT FILM	WITH FILM
WEIGHT LBS	300	770
MOM-INERTIA $I_{x'x'}$* LB FT^2	2.0×10^6	5.1×10^6
POLAR MOM INERTIA LB FT^2	3.7×10^6	9.2×10^6

* ABOUT AXIS NORMAL TO POLAR AXIS THROUGH THE CENTER OF THE LENS



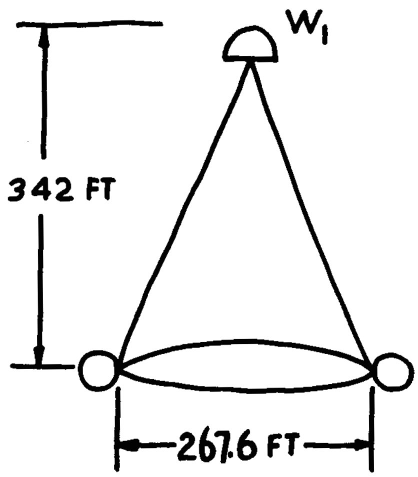


PRELIMINARY LOADS

SOURCES:

1. ORBITAL GER, 11277 & 11716
2. TORQUE COIL GER 11704
3. DAMPER GER 11816 APP U, V
4. SAIL GER 11816
5. PHOTOLYSIS OF FILM GER 11816, APP P

APPLICATION

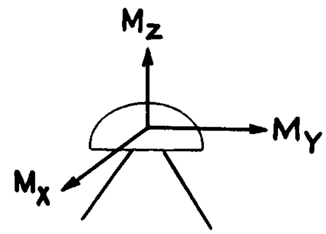


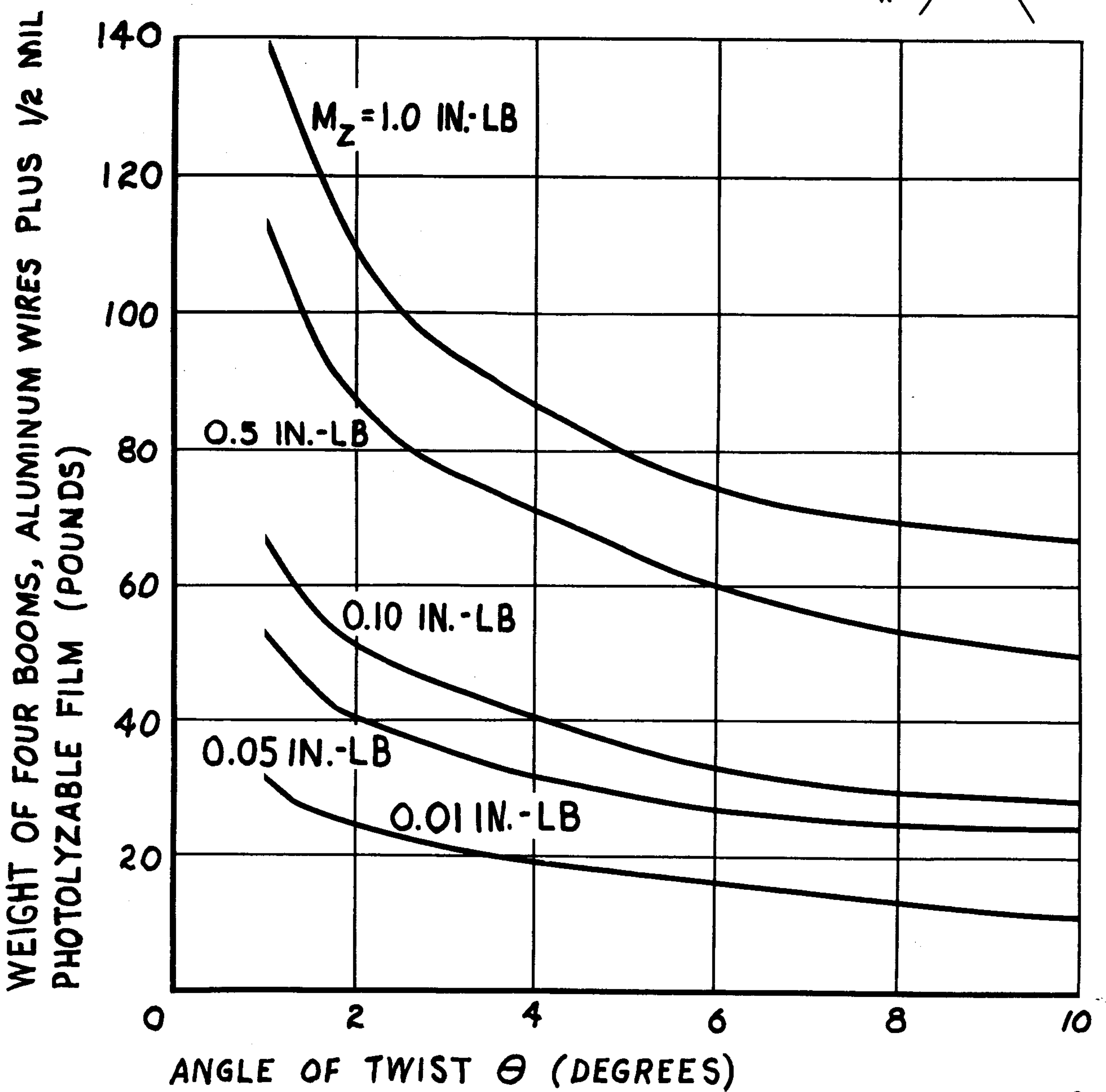
WEIGHT OF TETRAPOD = 12.5 LB

$W_1 = 400$ LB

LOAD SOURCE	CONDITION	BOOM NO.1 AXIAL $\times 10^3$	BOOM NO.1 TRANSVERSE $\times 10^3$
COIL	I	+0.244	—
	II	—	—
	III	—	—
	IV	—	-0.472‖y
	V	+0.043	+0.507‖n
	VI	—	-0.185‖y
	VII	+0.043	—
	VIII	—	-0.472‖x
	IX	+0.000184	—
SAIL	SOLAR PRESSURE HITS SATELLITE NORMAL TO THE SAIL	±0.722	—
GRAVITY GRADIENT	$\beta = 69°45'$ SAIL IN THE ORBITAL PLANE	±0.310	VERY SMALL
	$\beta = 69°45'$ SAIL NORMAL TO THE ORBITAL PLANE	0	»

POINT TORQUER





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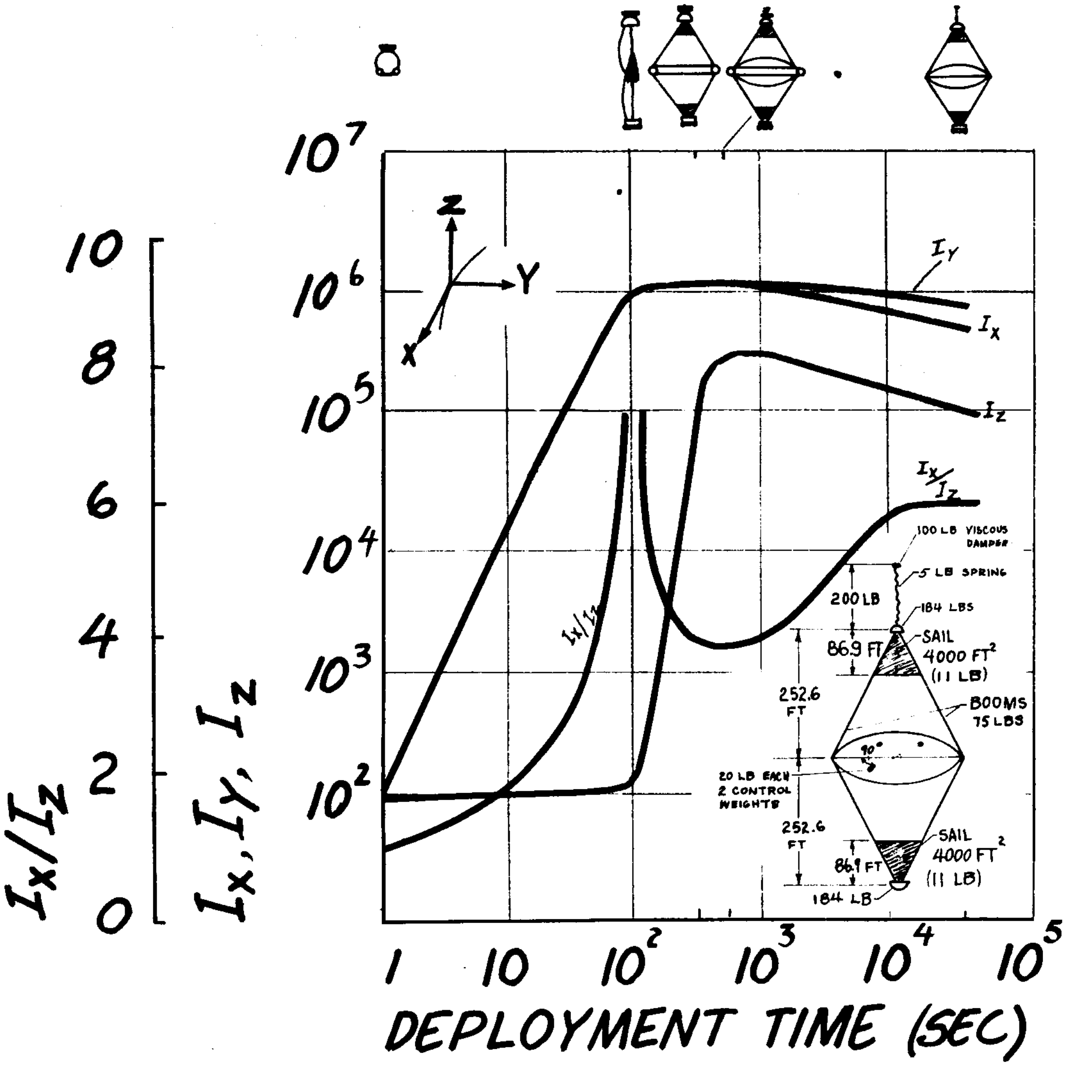
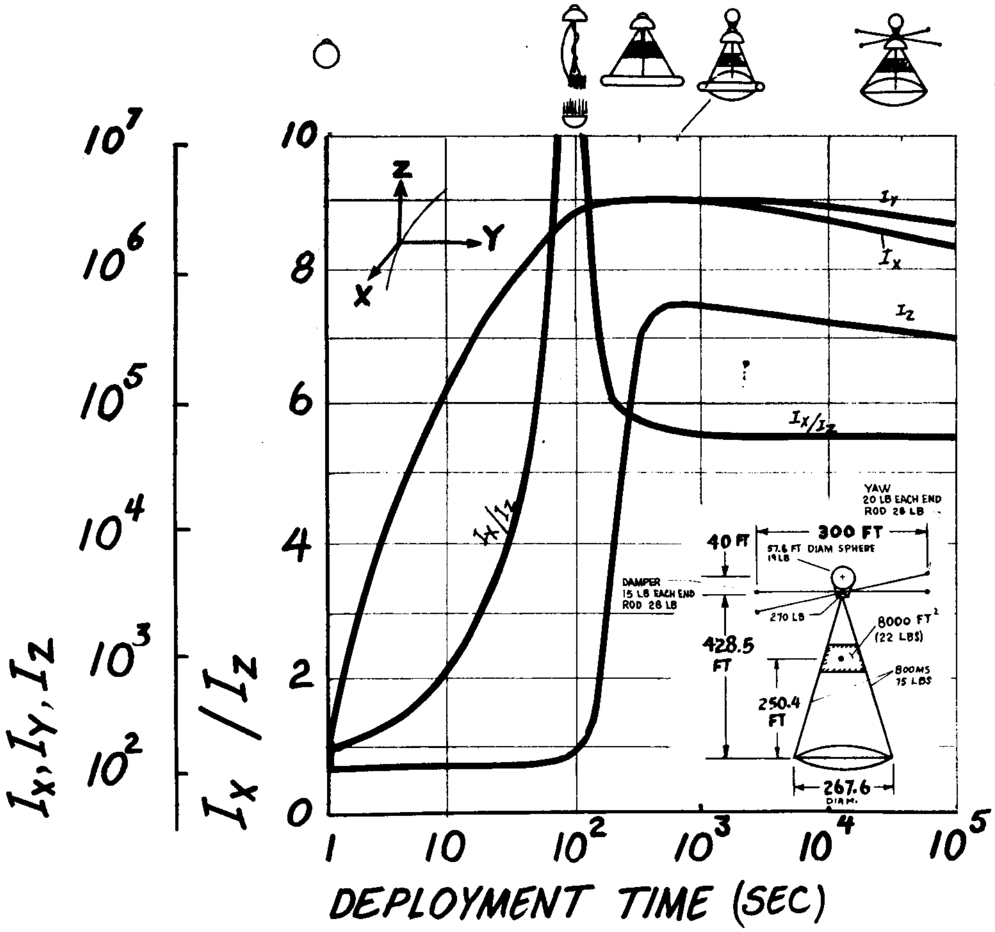
SYMMETRICAL SATELLITE INERTIA PARAMETERS DURING DEPLOYMENT



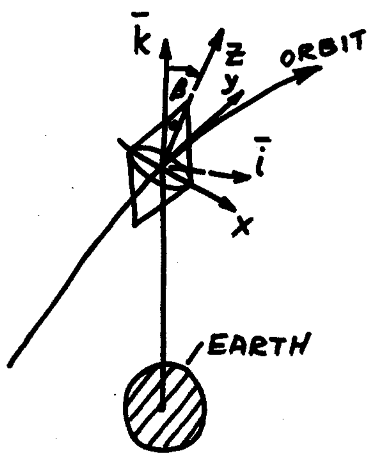
ASYMMETRIC SATELLITE INERTIA PARAMETERS DURING DEPLOYMENT



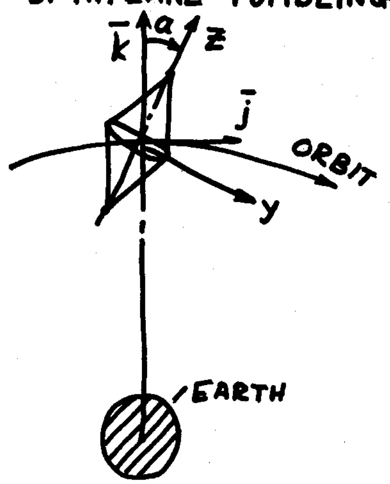
TUMBLING LOADS

CONDITIONS:

a. OUT OF PLANE TUMBLING

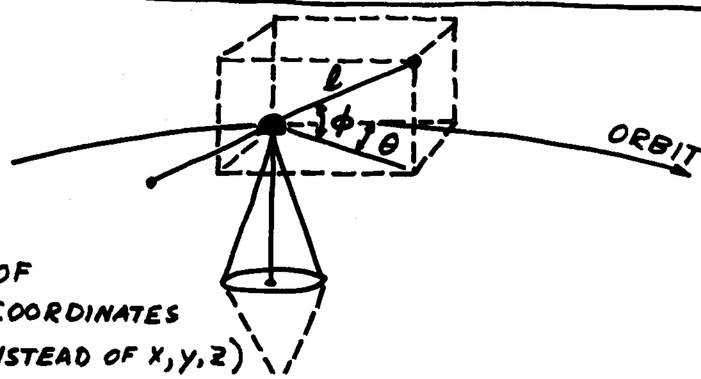


b. INPLANE TUMBLING



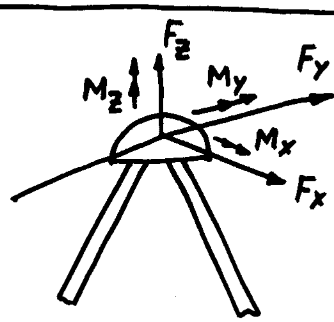
FORCES: $F\left(\beta, (\text{or } a), \omega_0, \omega, \frac{I_{xx}}{I_{zz}}, x, y, z\right)$

AMES DAMPER



FORCES:
FUNCTIONS OF
SPHERICAL COORDINATES
θ, ℓ, ϕ (INSTEAD OF x, y, z)

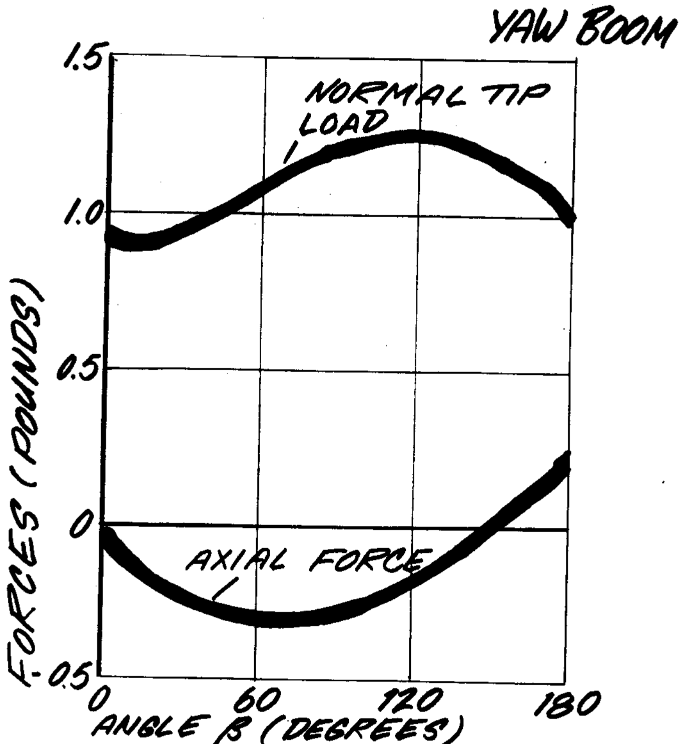
APEX

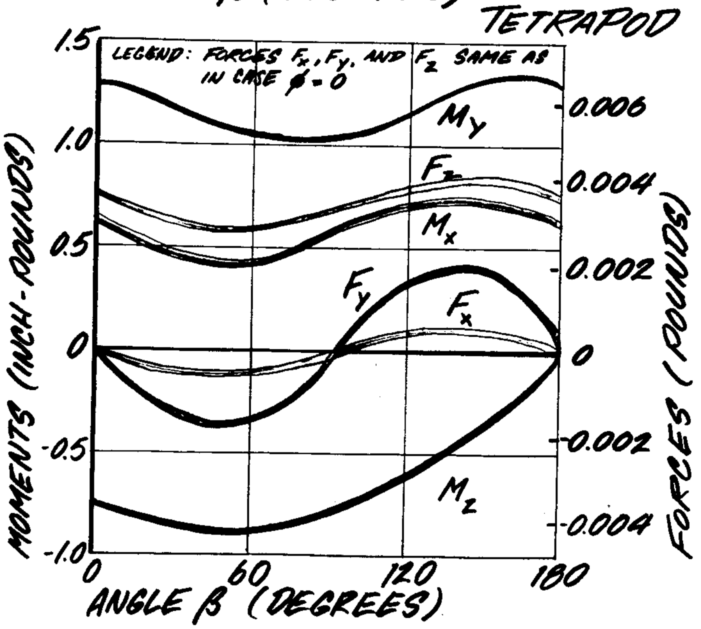


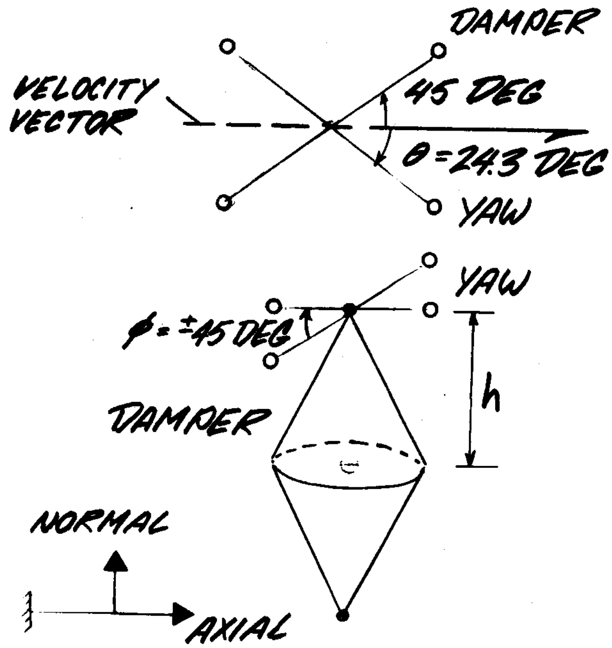
// TO ORBIT

FORCES & MOMENTS FROM DAMPER & YAW RODS AND FROM CANISTER CONCENTRATED MASS

TUMBLING APPLICATION







$\omega_{ROLL} = 4\omega_0$

$\beta \cong 80$ DEG

TUBE DIA. = 1 1/8 IN.

$\delta_{LOAD} = 35.1$ IN.

$\delta_{TEMP} \cong 40$ IN.

$\delta_{TOTAL} \cong 75$ IN.

$\omega_{ROLL} = 4\omega_0$

$\beta \cong 70$ DEG

WEIGHT OF 4 BOOMS = 75 LB

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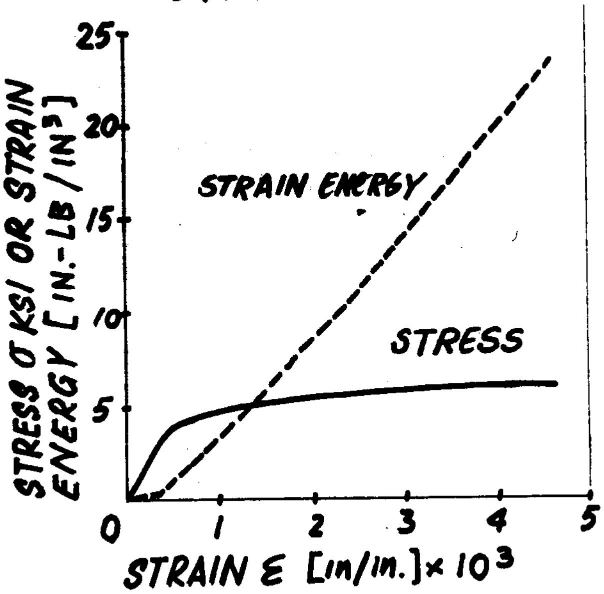
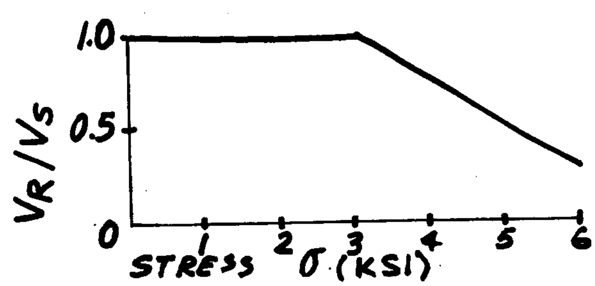
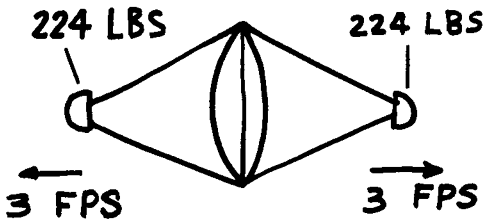
SEPARATION VELOCITY

STRAIN ENERGY



RECOVERY VELOCITY





ASYMMETRICAL

$KE = \frac{1}{2} m V_s^2 = 376$ in./lbs

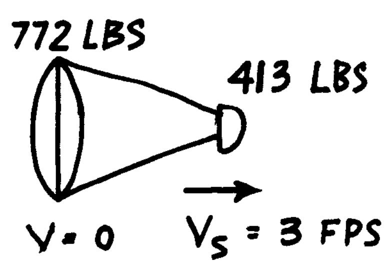
WIRE VOL 26.4 Cu IN.

BOOMS EFFECTIVE	STRAIN ENERGY DENSITY	MAX STRESS	V_R/V_S	RETURN VELOCITY (FPS)
1	14.23	5,500	0.38	1.14
3	4.75	5,000	0.52	1.56

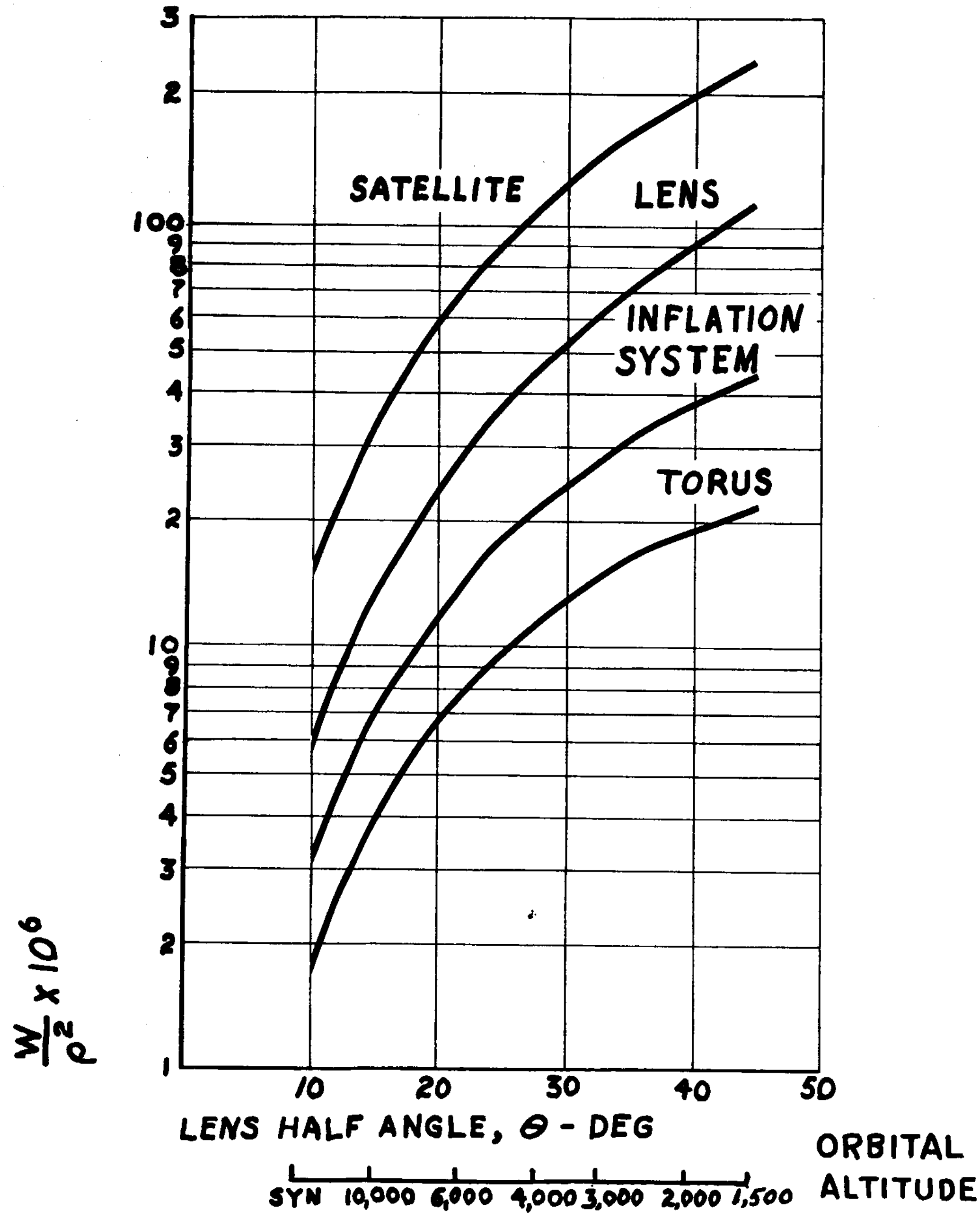
ASYMMETRICAL

$$\Delta E = \frac{1}{2} V_S^2 \frac{M_C M_L}{M_C + M_L}$$

$= 440$ IN. LBS



WEIGHT TRADE-OFF



WEIGHT - SAVING STUDY

ITEM	PARAMETER	G^2S^2 VALUE	PROJECTED VALUE	POTENTIAL WEIGHT SAVING
LENS MATERIAL	w	29.7×10^{-6}	12.97×10^{-6} LB/IN^2	321 LB
LENS MATERIAL	N	.3792 LB/IN	.0471 LB/IN	294 LB
TORUS MATERIAL	F_T/γ_T	$.263 \times 10^6$ IN	SAME	NONE
BOTTLE MATERIAL	F_B/γ_B	10^6 IN	1.8×10^6 IN	93 LB
INFLATION GAS	m	4	SAME	NONE
GEOMETRY	r/R	.02927	SAME	NONE
F.S. TORUS PRESSURE	a_1	1.25	1.10	37 LB
F.S. TORUS STRENGTH	a_2	1.25	SAME	NONE
INFLATION SYSTEM	a_3	1.12	SAME	NONE
F. S. BOTTLE	a_4	3.00	1.50	103 LB
GAS LEAK & RESERVE	a_5	3.04	2.00	77 LB

SOLAR SAILING AND STABILIZATION REQUIREMENTS

SOLAR SAILING

MOBILITY	100 DEG/MO.
MODES	BUILDUP/DECAY/STANDBY
YAW CONTROL	CONTINUOUS OR DISCRETE
TIME CONST.	25 ORBITS
TOLERANCE	10 DEG/30 DEG

STABILIZATION

ROLL AND PITCH

TIME CONST.	8 ORBITS
TOLERANCE	3 DEG
ECCENTRICITY BUILDUP	5 PERCENT MAX

INERTIA RATIO CONSIDERATIONS

PRINCIPAL TRADE

RESTORING TORQUES

VS

WEIGHT AND SIZE

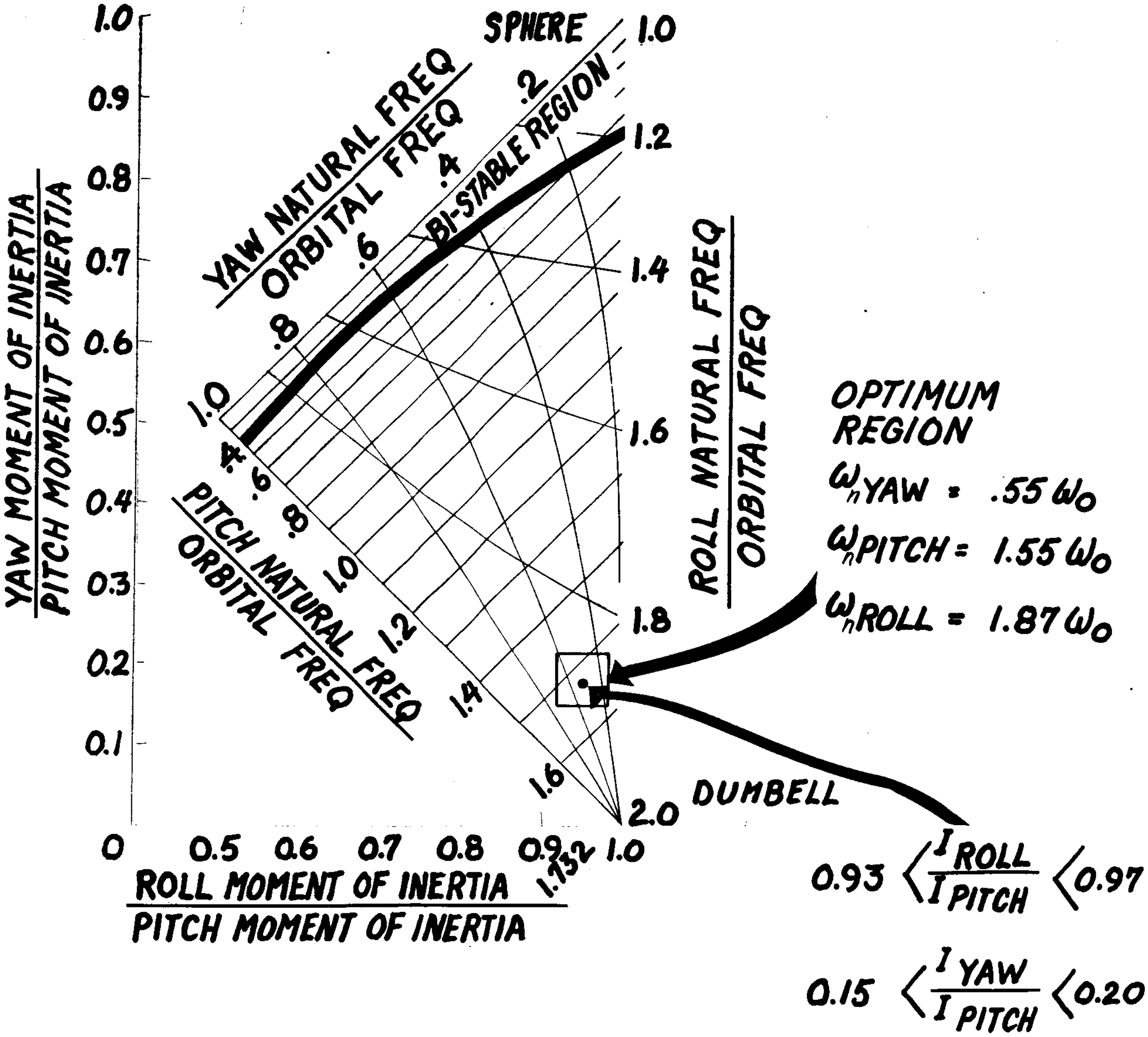
VS

PERTURBING FREQUENCIES

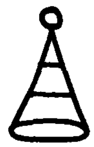
PERTURBING FREQUENCIES

SOLAR PRESSURE	$0, 1\omega_0, 2\omega_0$ ETC
ORBITAL ECCENTRICITY	$1\omega_0$
MAGNETIC FIELD	$1\omega_0, 2\omega_0, 3\omega_0$

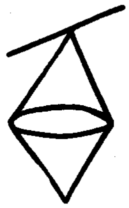
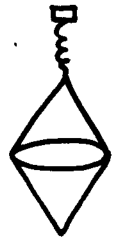
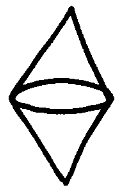
INERTIA RATIO NATURAL FREQUENCY MAP



INITIAL PITCH TUMBLING IMPULSES, LB-FT-SEC

CONFIGURATION → / PERTURBING SOURCE ↓	CASE 1  I_x = 960,000 I_y = 1,000,000 I_z = 122,000 TUMBLING IMPULSE = 861 LB-FT-SEC	CASE 2  I_x = 960,000 I_y = 1,000,000 I_z = 122,000 TUMBLING IMPULSE = 861 LB-FT-SEC	CASE 3  I_x = 1,920,000 I_y = 2,000,000 I_z = 122,000 TUMBLING IMPULSE = 1928 LB-FT-SEC
INITIAL PITCH ERROR RATE = $1\omega_o$	620	620	1240
INITIAL PITCH ERROR = 30°	430	430	960
YO-YO DESPIN UNCERTAINTY	66	66	66
INFLATION GAS ESCAPE	216/1080	288/1400	432/2160
SOLAR PRESSURE DURING PHOTOLYZATION	15	1100	1500
PHOTOLYZATION PARTICLE EJECTION	NEGLIGIBLE	NEGLIGIBLE	NEGLIGIBLE
ORBITAL ECCENTRICITY e = .02	30	30	20
ALGEBRAIC SUM OF IMPULSES	2241	3686	5946
MAXIMUM TUMBLING RATE RAD/SEC	$3.55\omega_o$	$5.9\omega_o$	$4.7\omega_o$
RSS OF IMPULSES	1320	1960	3060
PROBABLE TUMBLING RATE RAD/SEC	$2.15\omega_o$	$3.16\omega_o$	$2.47\omega_o$

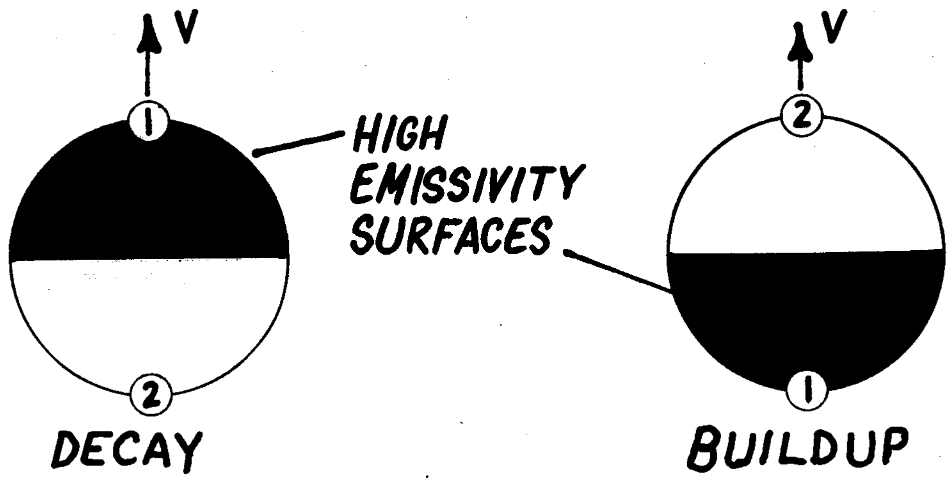
GRAVITY GRADIENT DAMPERS

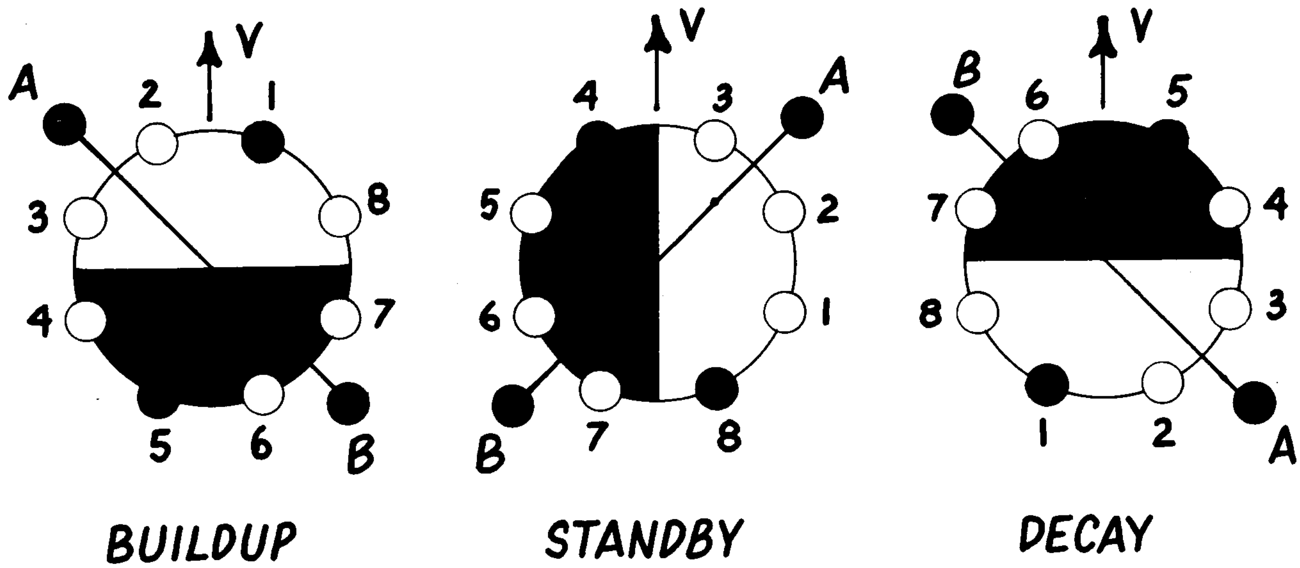
DAMPER → / PERFORMANCE ↓		AMES 	R/W 	MAG. HYS. 
DAMPING TIME CONSTANTS, ORBITS	VERT	5	4	20
	YAW	8	25	50
STEADY STATE ERRORS DEGREES	VERT	5°	3°	10°
	YAW	8°	15°	30°
UPRIGHT CAPTURE		DESIRABLE	DESIRABLE	NOT NEEDED
TUMBLE CAPABILITY		LIMITED	NIL	UNLIMITED
HIGH ALTITUDE CAPABILITY		SYNCH	SYNCH	LIMITED
COMPLEXITY		HIGH	MEDIUM	LOW
WEIGHT *		115 LB	150 LB	50 LB

* INCLUDES 50 LB OF WEIGHT TO ESTABLISH YAW STIFFNESS

DISCRETE YAW CONTROL

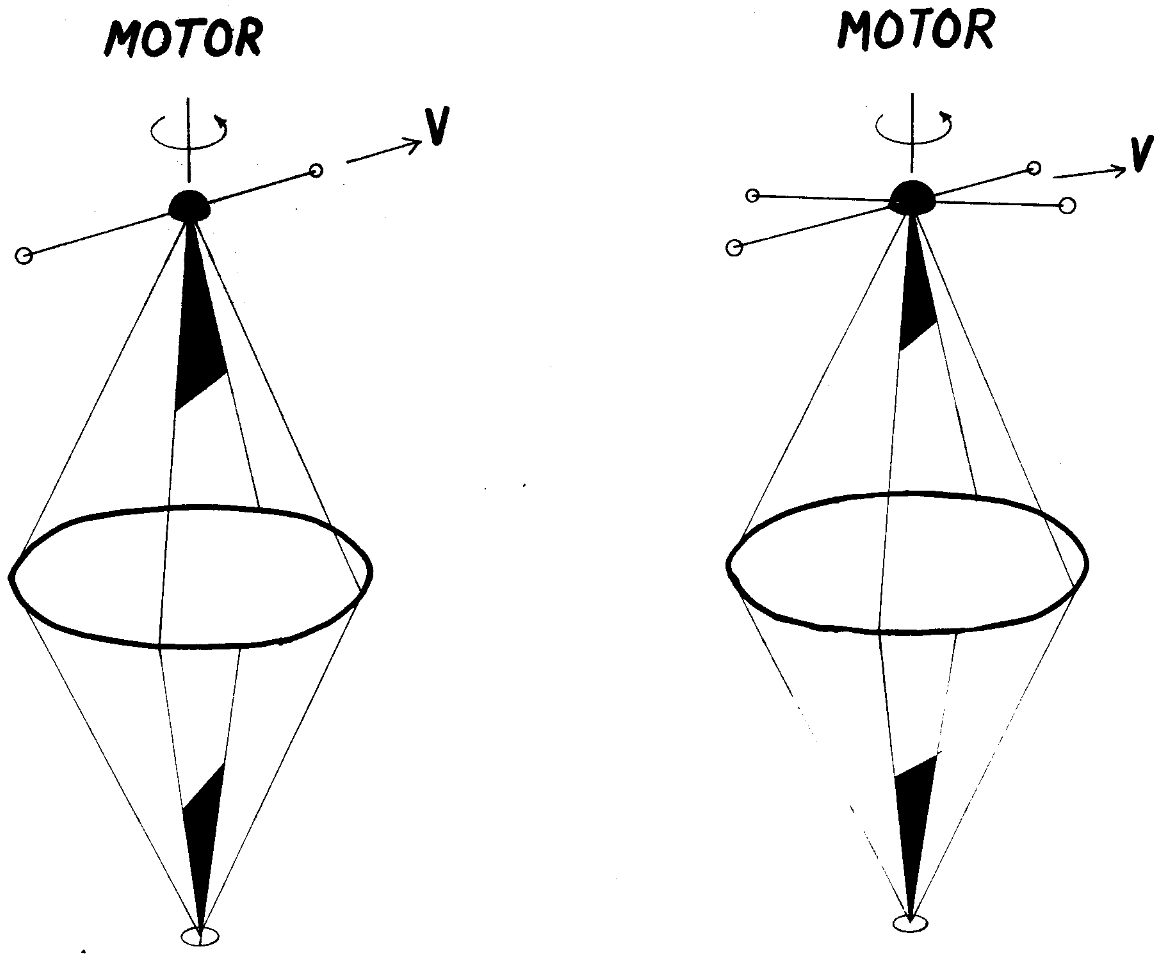
TWO MODE - FLYWHEEL - FIXED WEIGHTS





THREE MODE - MOVEABLE WTS - DAMPER BOOM

CONTINUOUS YAW CONTROL

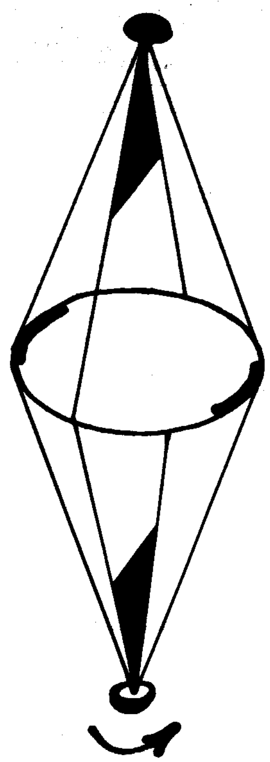


SINGLE YAW CONTROL BOOM

YAW BOOM WITH DAMPER BOOM

RECOMMENDED CONFIGURATIONS

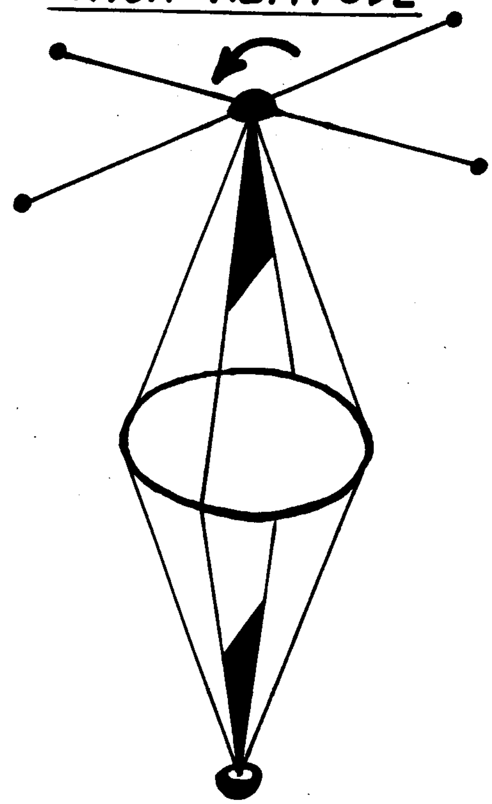
LOW ALTITUDE



DAMPER - HYSTERESIS

YAW CONTROL - FLYWHEEL

HIGH ALTITUDE



DAMPER - AMES

YAW CONTROL - BOOM DRIVE

CONCLUSIONS & RECOMMENDATIONS

CONCLUSIONS

GRAVITY GRADIENT STABILIZED LENTICULAR SATELLITE IS FEASIBLE

SOLAR SAILING OF LENTICULAR SATELLITE IS FEASIBLE

RECOMMENDATIONS

CONDUCT FURTHER SYSTEM STUDIES (NASA, IN-HOUSE, INDUSTRY) TREATING AREAS SUCH AS

- COST EFFECTIVENESS
- MULTIPLE ACCESS AND TERMINAL SHARING
- ADVANCED SATELLITES
- OPERATIONAL MODES
- GROUND ENVIRONMENT

DEFINE FLIGHT TEST PROGRAM

CONTINUE R AND D

- R-F PERFORMANCE
- MATERIALS, STRUCTURES, TOLERANCES
- STABILIZATION AND CONTROL
- STATION KEEPING
- GROUND MODELS AND TESTS

GOODYEAR AEROSPACE

GOODYEAR AEROSPACE CORPORATION

ENGINEERING MEMORANDUM REPORT

November 20, 1964
SM-8821

STUDY OF LENTICULAR SATELLITE WEIGHT

INTRODUCTION

A preliminary design for a gravity gradient stabilized lenticular satellite (G^2S^2) was presented in reference 1. This was designed for a 2000 NM circular orbit and had a lens radius of curvature of 200 feet and an included angle of 84°.

In the present study it has been established that a design (G^2S^4) incorporating solar sailing can be achieved for a modest increase in weight. In making this study it was assumed that the lens, rim, torus, and inflation system of the G^2S^2 design would also be satisfactory for the present study. This is a satisfactory approach to study the feasibility of adding solar sailing to the lenticular satellite. Having established feasibility and a preliminary design for a particular case, it is now advisable to study the effect of the various design parameters on the satellite launch weight.

The objectives of this study are:

1. To present satellite weight in a manner suitable for a system study.
2. To evaluate areas of potential weight reduction.

Derivation of Weight Equations

The major portion of the launch weight of the satellite, W_S, consists of three items: the lens, W_L; the torus, W_T; and the inflation system, W_I. The lens performs the primary function of the satellite, reflects microwave energy, and so its weight can be expressed in terms of the principal microwave parameters ρ and θ, the radius of curvature and half angle respectively. Furthermore, the torus and inflation system are only erection aids required to rigidize the lens and their weights must be functionally related to the lens weight. For the above reasons it is convenient to identify the sum of the weights of these three items as:

Eq. 1 $$W_P = W_L + W_T + W_I$$

From the weight statement for the full scale lenticular satellite appearing on page 23 of Reference 1

$$W_P = 552 + 117 + 233 = 902 \text{ lbs.}$$

$$W_S = 1250$$

For this particular case W_P accounts for 72% of the satellite weight. The remaining 28% is in items such as rim, damper system, canisters, etc., which will obey different sealing laws than W_P. It appears reasonable to assume that the weights of these remaining items will increase or decrease if W_P increases or decreases and it is suggested for a first approximation that

Eq. 2 $$W_S = b_1 W_p$$

where for this particular case

$$b_1 = \frac{1250}{902} = 1.385$$

Present studies concerned with incorporating solar sailing on the lenticular satellite are not complete but indicate that the weight increase will be modest and that W_P will still account for the major portion of the total launch weight. So for the advanced configuration it is suggested that

Eq. 3 $$W_S = b_2 W_P$$

where b_2 is to be determined at the conclusion of the current program.

Attention can now be focused on the determination of W_P. The geometry used is shown in Figure 1. The lens weight will be considered first. The total surface area is

Eq. 4 $$A_L = 4\pi\rho^2(1 - \cos\theta)$$

and multiplying by the unit weight of the lens material gives the total weight.

Eq. 5 $$W_L = 4\pi w\, \rho^2(1 - \cos\theta)$$

The torus area and weight may be written

Eq. 6 $$A_T = 4\pi^2 r(R + r)$$

Eq. 7 $$W_T = 4\pi\, \gamma_T\, rt(R + r)$$

For this study it is necessary to rewrite Equation 7 in terms of the basic parameters of the system. It should be noted that the torus must satisfy two criteria - wrinkling and strength, note page 106 of reference 1.

The torus loading, q, is radial and depends upon the strength level, N, required to rigidize the lens and is given by:

Eq. 8 $$q = 2N \cos \theta$$

The pressure required to satisfy the wrinkling criteria is given by the condition

$$a_1 q R = p_T \pi r^2$$

Eq. 9 $$p_T = \frac{2a_1 NR \cos \theta}{\pi r^2}$$

where a_1 is a factor of safety on the torus pressure.

The strength criteria is

Eq. 10 $$\frac{p_T r}{t} (2 + r/R) = \frac{F_T}{a_2}$$

where a_2 is a factor of safety on the strength of the torus.

Solving equations 9 and 10 for rt yields

Eq. 11 $$rt = \frac{a_1 a_2 N\rho(2 + r/R) \cos \theta \sin \theta}{\pi F_T}$$

and substituting the above equation into Equation 7 yields an expression for the torus weight in terms of the desired parameters.

Eq. 12 $$W_T = 4\pi a_1 a_2 N\rho^2 (\frac{\gamma_T}{F_T})(1 + r/R)(2 + r/R) \cos \theta \sin^2 \theta$$

The inflation system weight, W_I, consists of the sum of the gas, bottle, and hardware weight. From page 23 of reference 1 it is apparent that the hardware weight is small compared to the gas and bottle weight and so the inflation system weight may be written

Eq. 13 $$W_I = a_3(W_G + W_B)$$

where a_3 accounts for the hardware weight.

The gas weight is given by the standard equation

Eq. 14 $$W_G = \frac{mpV}{1545T \times 12} = \frac{mpV}{18,550T}$$

where the factor 12 is introduced to keep the length units in inches, and the weight of the bottle is given by

Eq. 15 $$W_B = \frac{3a_4}{2} \left(\frac{\gamma_B}{F_B}\right) pV$$

where a_4 is a factor of safety on strength of the bottle.

Substituting Equations 14 and 15 into Equation 13 gives for the total weight of the inflation system:

Eq. 16 $$W_I = a_3 \left(\frac{m}{18,550\ T} + \frac{3a_4}{2} \frac{\gamma_B}{F_B}\right) pV$$

The quantity pV must still be determined. Both the lens and torus contribute to this quantity and it may be written

Eq. 17 $$pV = p_T V_T + p_L V_L$$

The pressure in the torus is given by Equation 9 and the lens pressure is

Eq. 18 $$p_L = \frac{2\ N}{\rho}$$

The two volumes are given by

Eq. 19 $$V_T = 2\pi^2 r^2 \rho(1 + r/R) \sin \theta$$

Eq. 20 $$V_L = \frac{2}{3}\pi \rho^3(1 - \cos \theta)^2(2 + \cos \theta)$$

Substituting the above expressions into Equation 17 yields

Eq. 21 $$pV = 4\pi a_5 N\rho^2\left[a_1(1+\frac{r}{R})\cos\theta\ \sin^2\theta + \frac{1}{3}(1 - \cos\theta)^2(2 + \cos\theta)\right]$$

where a_5 is a factor to account for gas leakage and reserve.

Substituting the above expression into Equation 16 gives the inflation system weight in terms of the desired parameters.

Eq. 22 $$W_I = 4\pi a_3 a_5 N\rho^2\left(\frac{m}{18,500\ T} + \frac{3a_4}{2}\ \frac{\gamma_B}{F_B}\right)$$

$$\times\left[a_1(1 + r/R)\cos\theta\ \sin^2\theta + \frac{1}{3}(1 - \cos\theta)^2(2 + \cos\theta)\right]$$

The satellite weights have been derived in terms of the principal system parameters. The microwave parameters are ρ and θ. The material parameters are w, N, γ_T/F_T, γ_B/F_B, and m.

The reliability parameters are a_1, a_2, a_4, and a_5.

The equations required to calculate the weights are 1, 2, 3, 5, 12, and 22.

Check of Weight Equations

Before examining the weight equations in detail it is advisable to check them against the weights in reference 1. Only equations 5, 12, and 22 need be checked. All of the parameters for the lenticular satellite are listed in Table I along with the page in reference 1 from which they were obtained. Substituting these values in the appropriate equations gives

Eq. 23 $$W_L = 373.2 \times 10^{-6}\ \rho^2(1 - \cos\theta)$$

$$= 552.15 \text{ lbs.} \quad (552 \text{ Ref. 1 Value})$$

Eq. 24 $$W_T = 59.1 \times 10^{-6}\ \rho^2\ \cos\theta\ \sin^2\theta$$

$$= 113.29 \text{ lbs} \quad (116.6 \text{ Ref. 1 value})$$

Eq. 25 $$W_I = 16.23\times10^{-6}\rho^2(.4069+4.50)\ \ 1.2866\cos\theta\sin^2\theta + \frac{1}{3}(1-\cos\theta)^2(2+\cos\theta)$$

$$= 79.64\times10^{-6}\rho^2\ \ 1.2866\cos\theta\sin^2\theta + \frac{1}{3}(1-\cos\theta)^2 + \cos\theta)$$

$$= 224.02 \text{ lbs.} \qquad \text{(233 Ref. 1 value)}$$

The lens weight checks exactly and the torus and inflation system weights are about 3% low. This discrepancy is caused by the fact that in reference 1 the torus was conservatively assumed to be loaded at a radius of R + r instead of at a radius of R. Therefore the loads and the weight are high by a factor of 1 + r/R or 1.02927. This same factor also applies to the inflation system weight. It is concluded that the weight equations are correct.

Discussion of Microwave Parameters on Weight

There are many factors, such as; ground stations, orbits, etc., that must be considered in determining the optimum communication system. It is not the intent of this memornadum to study the effect of these factors on the satellite weight but rather to present the weight in terms of the microwave parameters that are defined by these factors. These microwave parameters are lens radius of curvature, ρ, and included angle, 2θ.

The equations developed above are in form suitable to determine the effect of the principal microwave parameters (ρ and θ) on the satellite weight. From Equations 1 and 2 the lenticular sagellite weight is

Eq. 26 $$W_S = 1.385\,(W_L + W_T + W_I)$$

where W_L, W_T and W_I are given by Equations 23, 24, and 25 respectively.

Inspection of these equations shows that the launch weights of the lens, torus, and inflation system are each proportional to the square of the lens radius. It follows from Equation 26 that the satellite weight is also proportional to the square of the lens radius. The general form of the weight equation is then

Eq. 27 $$W/\rho^2 = f(\theta).$$

The values of $f(\theta)$ have been calculated for 5 degree increments of θ covering a range from 10° to 45°. In addition 42° has been included because this is the angle used for the lenticular satellite. The values are shown in Table II for the lens, torus, inflation system, and satellite, and are plotted in Figure 2.

From Figure 2 it is apparent that the weight of the satellite depends strongly upon the angle θ. Comparing 10^o to 42^o for example

$$\frac{W_{10}}{W_{42}} = \frac{14.53}{213.87} = .068$$

Or a satellite with a 20^o included angle would weigh 6.8% of one with an 84^o included angle if the radius of curvature of the lens is the same.

The same curve is also useful for making a stabilization system trade-off study. The weight penalty for the satellite can be determined as a function of the increase in angle ($\Delta\theta$) and compared to the stabilization system weight required to limit the oscillations of the satellite to this value to arrive at the optimum arrangement. In this connection it is pertinent to note that the percent weight increase per degree decreases with the angle θ. From Figure 2 it is found that for

a. $\theta = 10^o$: $\Delta w = 21$ percent per degree

b. $\theta = 40^o$: $\Delta w = 4.2$ percent per degree

Another consideration of interest is the distribution of the weight between the lens, torus, and inflation system. This distribution can be readily obtained from Table II. The results are presented in Figure 3 as plots of percent of satellite weight against θ. The percentage of lens weight increases with θ and both the torus and inflation system percentages decrease.

Evaluation of Design Parameters

An examination of the weight equations derived previously reveals that in addition to the microwave parameters ρ and θ there are many other parameters that affect the launch weight of the satellite. These other parameters are designated design parameters, herein. Each of these design parameters will now be examined and areas of potential weight reduction discussed.

a. Lens Material

The lens weight is proportional to w (Eq. 5) and the torus and inflation system weights are proportional to N (Eqs. 12 and 22). From this observation it is apparent that of all the design parameters the lens material properties has the greatest influence on the satellite weight. For this reason the lens material properties deserve special attention.

The parameters w and N depend upon the wire and film properties and dimentions. They are given by

Eq. 27 $$w = \frac{\pi d^2}{2S} \gamma_w + t \gamma_F$$

Eq. 28 $$N = \frac{\pi d^2}{4S} F_y$$

For the G^2S^2 design the lens material consisted of one mil copper wire at a spacing of 1/21 inches with a yield stress of 23,000 psi in combination with 1/2 mil of photolyzable film. The values for this material are

$$w = \frac{\pi(.001)^2 \times .21 \times .324}{2} + .0005 \times .038$$

$$= (10.7 + 19) \times 10^{-6}$$

$$= 29.7 \times 10^{-6} \quad lb/in^2$$

$$N = \frac{\pi(.001)^2 \times 21 \times 23{,}000}{4}$$

$$= .3793 \quad lb/in.$$

It is interesting to note that the film weight is 19 x 100/29.7 or 64.1% of the lens weight and accounts for 354 lbs of the launch weight. The density of the film material cannot be changed but the possibility of decreasing the film thickness should be investigated.

The wire material and dimensions were dictated primarily by weaving limitations. Further effort may be warrented to increase S and to decrease F_y of the copper cloth or even better to develop technique for weaving with small diameter aluminum wire. Another approach is to explore the possibility of using filament wound material in which small diameter aluminum wire is possible.

It may be optimistic but perhaps possible to develop a lens material consisting of 1 mil aluminum wire at 1/10 inch spacing with a yield strength of 6,000 psi in combination with .3 mil photolyzable film. If this could be done then the lens material parameters would be

$$w^* = \frac{\pi(.001)^2 \times .1}{\times .1} + .0003 \times .038$$

$$= (1.57 + 11.40) \quad 10^{-6}$$

$$= 12.97 \times 10^{-6} \quad lbs/in^2$$

$$N^* = \frac{\pi(.001)^2}{4 \times .1} 6{,}000 = .0471 \text{ lbs/in}$$

With the above properties the lens weight would reduce from 552 lbs to 12.97 x 552/29.7 or 231 lbs and the torus plus inflation system weight from 337 lbs to .0471 x 337/.3793 or 42 lbs. The satellite weight would then be only 273 x 100/889 or 30.9% of the G^2S^2 weight.

b. **Torus Material**

The torus weight is proportional to γ_T/F_T (Eq. 12) and does not affect the lens or inflation system weights. At this time it does not appear feasible to decrease γ_T or increase F_T in an effort to improve this ratio. Furthermore since the torus weight is the smallest of the three that make up W_P, improvements in torus material will have a correspondingly small effect on the total weight.

c. Bottle Material

The inflation system weight is proportional to the sum of the gas and bottle weights and the bottle weight is proportional to γ_B/F_B (Eq. 22). Titanium was used for the bottle and has a value of F_B/γ_B of 10^6 in. Literature from manufacturers indicate that values of 1.8×10^6 may be attainable with fiberglass bottles. Using $\gamma^*_B/F^*_B = .555 \times 10^{-6}$ and Equation 25, the new weight would be only

$$\frac{(4.50/1.8 + .41) \times 100}{4.50 + .41} = 59.2\%$$

of the G^2S^2 inflation system weight or a reduction in weight of 224 x .418 = 93 lbs.

d. Inflation Gas

The weight of the inflation gas is proportional to the molecular weight, m. For helium, m = 4, and the only gas with a lower molecular weight is hydrogen with m = 2. This would reduce the gas weight in half but since the gas weight is small the weight savings possible does not appear to compensate for the increase in danger associated with hydrogen.

e. Torus Radius to Lens Radius (r/R)

This factor affects both the torus weight (Eq. 12) and the inflation system weight (Eq. 22). A value of .02927 was found satisfactory for G^2S^2 and perhaps a smaller value could be justified. However, the torus weight is proportional to $(1 + 3r/R)$ and the inflation system proportional to something less than $(1 + r/R)$ and so at most a 10% and 3% reduction for the two items is possible. This is not sufficient to warrant a change in r/R.

f. Factor a_1

This is a factor to insure that the torus pressure is always greater than that required to support the membrane loads of the lens. It has an effect on both the torus and inflation system weights. This was rather arbitrarily chosen as 1.25. A smaller value may be acceptable, perhaps

$$a^* = 1.10$$

The new weight would be $(113 + 224)1.10/1.25 = 300$ lbs. or a weight saving of 37 lbs.

g. Factor a_2

This is a factor of safety against the strength of the torus and affects only the torus weight. A value of 1.25 was used and is felt to be a minimum consistent with good structural reliability. A reduction in a_2 is not advisable.

h. Factor a_3

This factor is simply the ratio of the inflation system weight to the weight of the gas plus bottle. The value of 1.12 was determined from the weights used on G^2S^2 and there is no apparent reason to expect it to become smaller.

i. Factor a_4

This is a factor of safety against the strength of the bottle and affects only the weight of the inflation system. Since the bottle weight is large compared to the gas weight, it has considerable effect. A factor of 3 was used for G^2S^2 which is undoubtedly conservative. If this were reduced to

$$a^*_4 = 1.5$$

the new weight would be $224(.41 + 2.25)/(.41 + 4.50) = 121$ lbs., a saving of 103 lbs.

j. Factor a_5

This factor accounts for gas leakage and reserve and the inflation system weight is directly proportional to it (Eq. 22). The value determined for G^2S^2 was 3.04. There are two factors that warrant further study in an effort to reduce leakage. There are the hole area assumed in the torus and the time that the design pressure is maintained. Both of these probably can be reduced and if the leakage and reserve were cu in half the factor would be

$$a^*_5 = 2.00$$

and the new weight would be 224 x 2/3.04 = 147 lbs. or a saving of 77 lbs.

Each of the design parameters have been examined, potential improvements discussed, and the corresponding weight savings estimated. This is summarized in Table III. The column, G^2S^2 values, are the values used in reference 1 for the full scale satellite. The column, projected values, are perhaps optimistic values of the design parameters that may be realized. The last column is the weight savings associated with the change of the particular parameter.

From this table it is apparent that a large reduction in launch weight may be possible. The savings, it should be noted, is not the sum of the weight savings shown because these parameters enter in most cases as products rather than sums. The sum of the weight savings shown in the table is 926 lbs whereas if all of the projected design parameters were used the corresponding weight savings would be 642 lbs. The corresponding satellite weight would be 360 lbs or 28.8% of the G^2S^2 weight.

The largest weight reduction, 614 lbs, is associated with the lens material. This may be considered undully optimistic, however, it does demonstrate the importance of the lens material properties on the weight and that efforts to improve these properties should be given serious consideration.

The next largest weight reduction is in the gas bottle. Substantial savings are possible (103 lbs) by reducing the factor of safety from 3 to 1.5 or (93 lbs) by using improved materials with the same factor of safety. A more detailed study of the bottle should be made to select the best combination of material and factor of safety required to achieve the desired level of reliability.

Third largest weight saving (77 lbs) is associated with the factor a_5 which takes into account gas leakage and reserve.

The number of programmed holes in the torus for packaging and the time required at design pressure should be investigated to determine whether this factor can be reduced.

The smallest weight savings (37 lbs) is associated with the factor a_1, the factor of safety on the torus design pressure.

Summary and Conclusions

1. Equations have been developed for predicting the launch weight of a lenticular satellite.

2. The weight of a satellite based on G^2S^2 design parameters as function of the microwave parameters ρ and θ is shown in Figure

3. The effect of the design parameters on the satellite weight is discussed and potential weight savings shown in Table III.

4. That the weight of G^2S^2 could by additional study and development be reduced from 1250 lbs to 360 lbs.

5. That prior to a system study the design parameters be reviewed and values compatible with the development effort anticipated be specified.

E. Rottmayer

E. Rottmayer

ER:ao

References

1. GER-11502 Feasibility Study and Preliminary Design of Gravity-Gradient-Stabilized Lenticular Test Vehicle.

SYMBOLS

A	in^2	Area
a		Factor, defined in text
b		Factor, deefined in text
d	in	Wire diameter
F	psi	Allowable stress
m		Molecular weight of gas
N	lbs/in	Yield load of lens material
p	psi	Pressure
q	lbs/in	Torus loading
r	in	Torus radius
R	in	Satellite radius
S	in	Wire spacing
t	in	Film thickness
T	^oR	Temperature
V	in^3	Volume
w	lbs/in^2	Unit weight of lens material
W	lbs	Weight
γ	lbs/in^3	Density
ρ	in	Radius of curvature of lens
θ	degrees	Lens half angle

Subscripts

B	Bottle
G	Gas
I	Inflation system
L	Lens
P	Lens + torus + inflation system
S	Satellite
T	Torus
1,2,3,4,5	Defined in text

TABLE I

G^2S^2 PARAMETERS

Item	Value	Ref. Pg.
ρ	2400 in	6
θ	42°	6
w	29.7×10^{-6} lb/in^2	149
N	.3793 lb/in	103
γ_T	.038 lb/in^3	149
F_T	10,000 lb/in^2	105
γ_B	.16 lb/in^3	Titanium
F_B	160,000 lb/in^2	Titanium
m	4	Helium
T	530° R	612
r/R	.02927	108
a_1	1.25	107
a_2	1.25	107
a_3	1.12	Note 1
a_4	3	Note 2
a_5	3.04	Note 3

Note 1. From page 23 $W_I = 233$ lbs

$$W_B + W_G = 208 \text{ lbs}$$

$$a_3 = 233/208 = 1.12$$

Note 2 Bottle weight calculation not shown, a factor of safety of 3 was used.

Note 3. Weight of helium to inflate lens and torus computed to be 5.60 lbs. Actual weight required including leakage 13.01 lbs. (Ref. Pg. 128) and 17 lbs (Ref. Pg. 23) was used to be conservative. $a_5 = 17/5.60 = 3.04$.

TABLE II

Weight as a Function of θ

θ	$W/\rho^2 \times 10^6$				
	Lens	Torus	Inflation System	L+T+I	Satellite
10	5.67	1.76	3.06	10.49	14.53
15	12.72	3.83	6.72	23.27	32.23
20	22.51	6.50	11.55	40.56	56.18
25	34.97	9.57	17.26	61.80	85.59
30	50.00	12.80	23.55	86.35	119.59
35	67.49	15.93	30.06	113.48	157.17
40	87.31	18.71	36.45	142.47	197.32
42	95.86	19.67	38.89	154.42	213.87
45	109.31	20.90	42.39	172.60	239.05

TABLE III

Summary of Parameter Evaluation

Item	Parameter	G^2S^2 Value	Projected Value	Potential Weight Saving
Lens Material	w	29.7×10^{-6}	12.97×10^{-6} lb/in^2	321 lbs
Lens Material	N	.3792 lb/in	.0471 lb/in	294 lbs
Torus Material	F_T/γ_T	$.263\times10^6$ in	Same	None
Bottle Material	F_B/γ_B	10^6 in	1.8×10^6 in	93 lbs
Inflation Gas	m	4	Same	None
Geometry	r/R	.02927	Same	None
F.S. Torus Pressure	a_1	1.25	1.10	37 lbs
F.S. Torus Strength	a_2	1.25	Same	None
Inflation System	a_3	1.12	Same	None
F.S. Bottle	a_4	3.00	1.50	103 lbs
Gas Leak & Reserve	a_5	3.04	2.00	77 lbs

LENTICULAR GEOMETRY

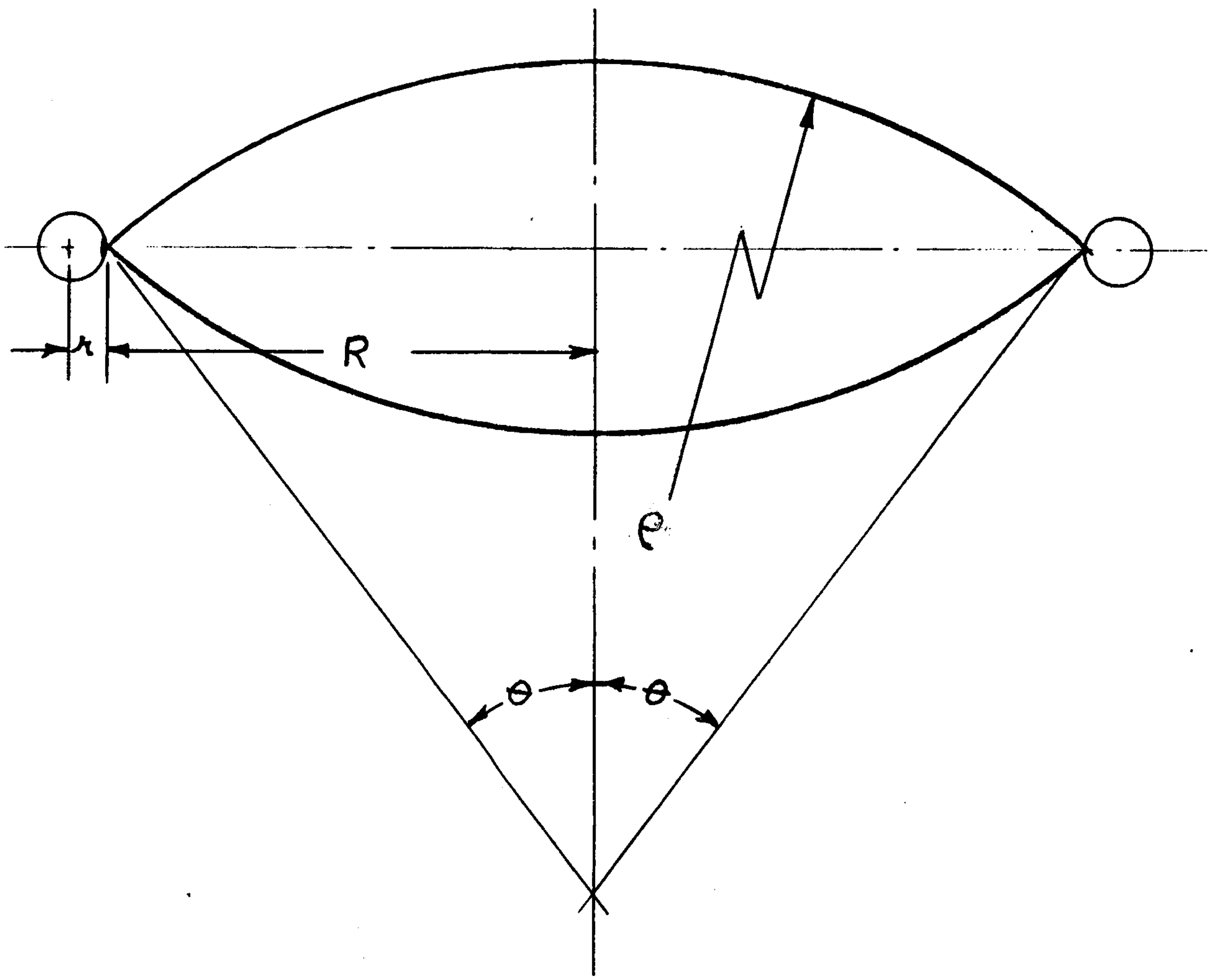


FIG 1

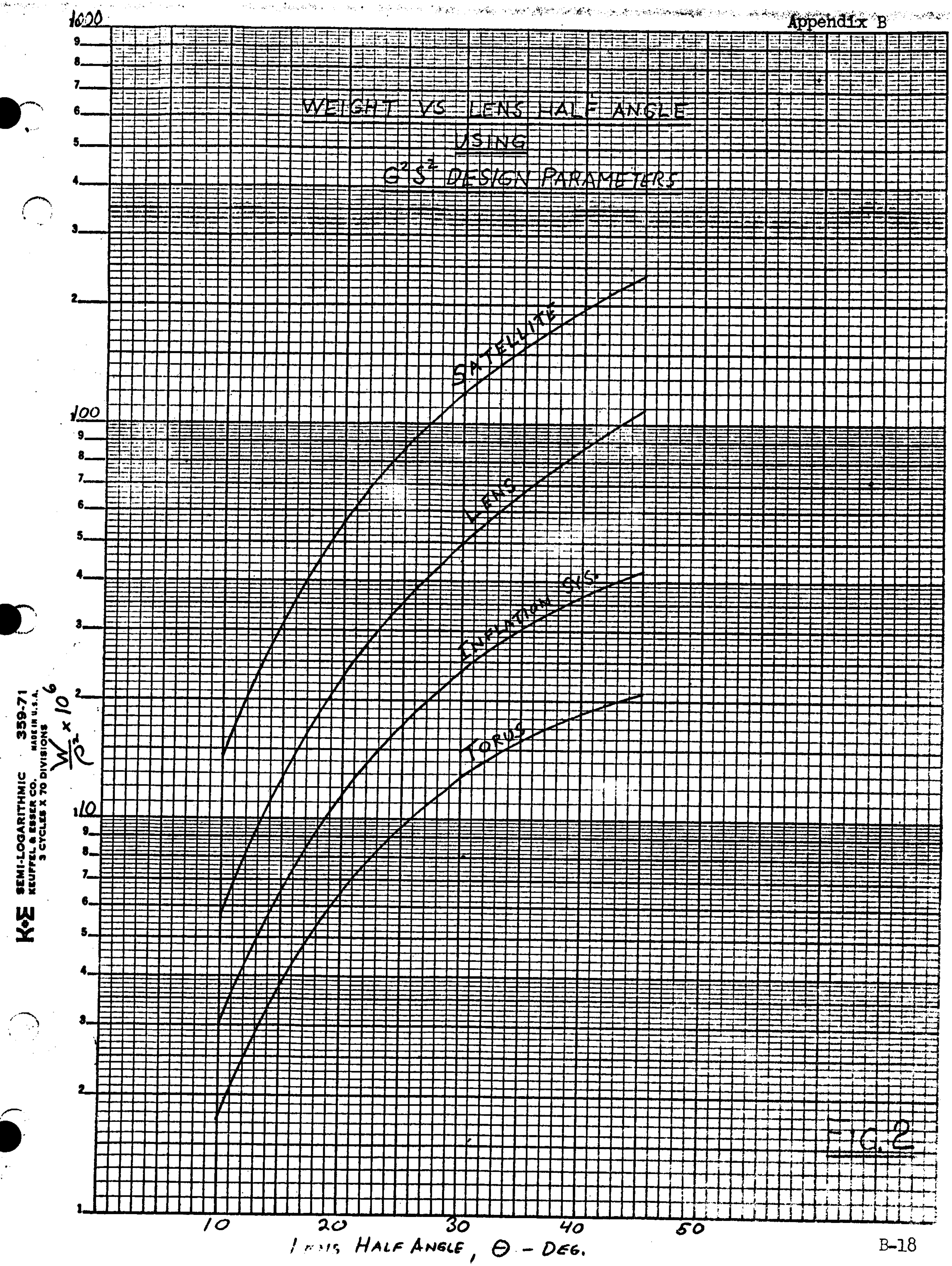
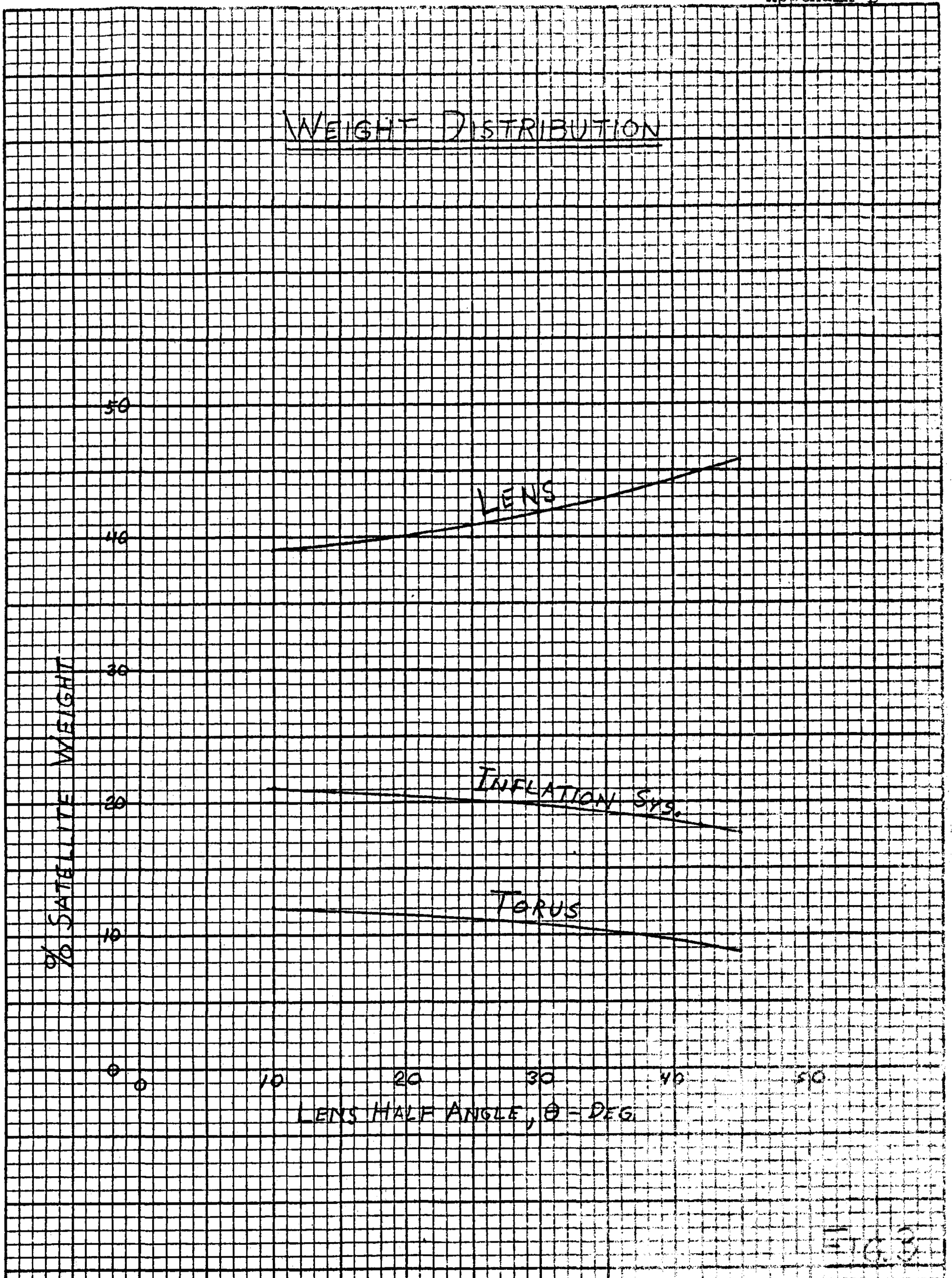

WEIGHT VS LENS HALF ANGLE
USING
G^2S^2 DESIGN PARAMETERS
SATELLITE
LENS
INFLATION SYS.
TORUS
$\frac{W}{\rho^2} \times 10^6$
1000
100
10
1
10
20
30
40
50
LENS HALF ANGLE, Θ - DEG.
K+E SEMI-LOGARITHMIC 359-71
KEUFFEL & ESSER CO. MADE IN U.S.A.
3 CYCLES X 70 DIVISIONS

FIG. 2


WEIGHT DISTRIBUTION
LENS
INFLATION SYS.
TORUS
% SATELLITE WEIGHT
50
40
30
20
10
0
0
10
20
30
40
50
LENS HALF ANGLE, θ - DEG
FIG. 3

GOODYEAR AEROSPACE CORPORATION

ENGINEERING MEMORANDUM REPORT

DECEMBER 1, 1964
SM-8827

Subject: TUMBLING SATELLITE.

References:

1. Rottmayer, E. and Marketos, J.D. Study of Orbital Design Conditions for a Gravity Gradient Stabilized Lenticular Satellite, Goodyear Aerospace Corporation GER 11277, Oct. 1, 1963.

2. Marketos, J.D., Preliminary Investigation of the Effect of Bending Moments M_x, M_y, M_z Applied at the Tetrapod Apex, on the Weight of the Tetrapod booms. Goodyear Aerospace Corporation Engineering Memorandum Report SM-8781, Sept. 28, 1964.

3. Advanced Passive Communications Lenticular Satellite Studies. Goodyear Aerospace Corporation Contract NAS 1-3114, Summary report, Phase II GER 11816 Nov. 1964.

Nov. 19/64

Appendix C

TUMBLING SATELLITE.

A. Rotation about pitch axis

Coordinate convention: y-axis tangent to the orbit for normal flight.

ω: Satellite angular velocity, vector along pitch axis.
ω_0: " " " in its orbit

Condition: $\omega = 4\omega_0$ ($\bar{\omega}$ pointing along the $-x$ axis)
Let at zero time the satellite be in the normal position.

Then, at time t,

$$\alpha = \omega t$$

1. Gravity gradient forces (Eqs 5 of Reference 1).

$$dF_{x1} = -\omega_0^2 x\,dm$$
$$dF_{y1} = -\omega_0^2(3z\sin\alpha\cos\alpha - 3y\sin^2\alpha)\,dm$$
$$dF_{z1} = -\omega_0^2(3y\sin\alpha\cos\alpha - 3z\cos^2\alpha)\,dm.$$

2. Inertia forces (See Eqs 6 of Reference 1)

$$dF_{y2} = +3\omega_0^2 y\left(\frac{I_y - I_z}{I_y}\right)\sin\alpha\cos\alpha\,dm = +\frac{3}{2}\omega_0^2 y\lambda\sin 2\alpha\,dm$$
$$dF_{z2} = -\frac{3}{2}\omega_0^2\lambda z\sin 2\alpha\,dm \qquad \left(\lambda = \frac{I_y - I_z}{I_y} \simeq 5/6\right)$$

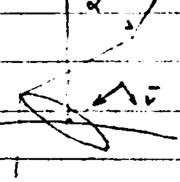
3. Centrifugal forces due to ω rotation

$$dF_{x3} = 0$$
$$dF_{y3} = \omega^2 y\,dm$$
$$dF_{z3} = \omega^2 z\,dm$$

4. Coriolis forces:

Consider the linear velocity v of the points of satellite due to its ω-rotation; then the coriolis acceleration as a vector is $2\bar{\omega}_0 \times \bar{v}$

Because $\bar{\omega}_0 \perp \bar{v}$ the magnitude of this acceleration is



$2|\bar{\omega}_0||\bar{v}|$, Hence

$$dF_{x4} = 0$$
$$dF_{y4} = -2\omega_0\omega y\,dm$$
$$dF_{z4} = -2\omega_0\omega z\,dm.$$

Resultant x, y and z forces.

$$
\left.\begin{aligned}
dF_x &= -\omega_0^2 x\, dm \\
dF_y &= \left[-3\omega_0^2\left(z \sin\alpha\cos\alpha - y\sin^2\alpha\right) + \omega^2 y - 2\omega\omega_0 y + \tfrac{3}{2}\omega_0^2 \lambda y \sin 2\alpha\right] dm \\
dF_z &= \left[-3\omega_0^2\left(y \sin\alpha\cos\alpha - z\cos^2\alpha\right) + \omega^2 z - 2\omega\omega_0 z - \tfrac{3}{2}\omega_0^2 \lambda y \sin 2\alpha\right] dm
\end{aligned}\right\} \quad (1)
$$

For $\omega = 4\omega_0$ Equations (1) become,

$$
\left.\begin{aligned}
dF_x &= -\omega_0^2 x\, dm \\
dF_y &= \omega_0^2\left[-3\left(z \sin\alpha\cos\alpha - y\sin^2\alpha\right) + 8y + \tfrac{3}{2}\lambda y \sin 2\alpha\right] dm \\
dF_z &= \omega_0^2\left[-3\left(y \sin\alpha\cos\alpha - z\cos^2\alpha\right) + 8z - \tfrac{3}{2}\lambda z \sin 2\alpha\right] dm
\end{aligned}\right\}
$$

or

$$
\left.\begin{aligned}
dF_x &= -\omega_0^2 x\, dm \\
dF_y &= \tfrac{3}{2}\omega_0^2 y\, dm \left[6{,}3333 + \left(\lambda - \frac{z}{y}\right)\sin 2\alpha - \cos 2\alpha\right] \\
dF_z &= \tfrac{3}{2}\omega_0^2 z\, dm \left[6{,}3333 - \left(\lambda + \frac{y}{z}\right)\sin 2\alpha + \cos 2\alpha\right]
\end{aligned}\right\} \quad (2)
$$

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Appendix C

B. Rotation about roll axis (ω̄-vector along the +y-axis)

$$\omega = 4\omega_0$$

$$\beta = \omega t$$

For half a rotation $t = \frac{T}{8} = \frac{\pi}{4\omega_0}$ (T = Satellite period).

1. Gravity gradient forces (Equations 14 of Reference 1)

$$dF_{x1} = -\omega_0^2\left[x(1-4\sin^2\beta) + 4z\sin\beta\cos\beta\right]dm$$

$$dF_{y1} = 0$$

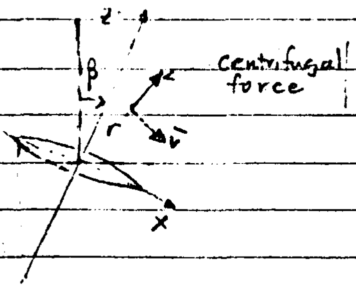
$$dF_{z1} = -\omega_0^2\left[4x\sin\beta\cos\beta + z(1-4\cos^2\beta)\right]dm$$

2. Inertia forces (See Eq. 15 of Reference 1)

$$dF_{x2} = +2x\omega_0^2\lambda\sin 2\beta\, dm$$

$$dF_{z2} = -2z\omega_0^2\lambda\sin 2\beta\, dm$$

3. Centrifugal forces due to ω-rotation



$$dF_{x3} = \omega^2 x\, dm$$

$$dF_{y3} = 0$$

$$dF_{z3} = \omega^2 z\, dm$$

4. Coriolis forces

The coriolis acceleration vector is $2\bar{\omega}_0 \times \bar{v}$

where $|\bar{v}| = r\omega$. This vector is along the roll (y) axis and points to the negative direction

The angle between $\bar{\omega}_0$ and $\bar{v}$ is $180° - \left(\beta + \tan^{-1}\frac{x}{z}\right)$

Hence,

$$dF_{x4} = 0$$
$$dF_{y4} = -2\omega\omega_0 (x^2+z^2)^{1/2} \cdot \sin\left(\beta + \tan^{-1}\frac{x}{z}\right) dm \quad (*)$$
$$dF_{z4} = 0.$$

The resultant x, y and z forces are then,

$$\left.\begin{aligned} dF_x &= \left[-\omega_0^2\left\{x(1-4\sin^2\beta) + 4z\sin\beta\cos\beta\right\} + \omega^2 x + 2x\omega_0^2\lambda\sin 2\beta\right] dm \\ dF_y &= -2\omega\omega_0(x^2+z^2)^{1/2}\sin\left(\beta+\tan^{-1}\frac{x}{z}\right) dm = -2\omega\omega_0(z\sin\beta + x\cos\beta)\,dm \\ dF_z &= \left[-\omega_0^2\left\{4x\sin\beta\cos\beta + z(1-4\cos^2\beta)\right\} + \omega^2 z - 2z\omega_0^2\lambda\sin 2\beta\right] dm \end{aligned}\right\} \quad (3)$$

For $\omega = 4\omega_0$ Equations (4) become,

$$dF_x = \omega_0^2\left[(15+4\sin^2\beta)x - 4z\sin\beta\cos\beta + 2x\lambda\sin 2\beta\right] dm$$
$$dF_y = -8\omega_0^2(x^2+z^2)^{1/2}\sin\left(\beta+\tan^{-1}\frac{x}{z}\right) dm = -8\omega_0^2(z\sin\beta + x\cos\beta)\,dm$$
$$dF_z = \omega_0^2\left[(15+4\cos^2\beta)z - 4x\sin\beta\cos\beta - 2z\lambda\sin 2\beta\right] dm$$

or

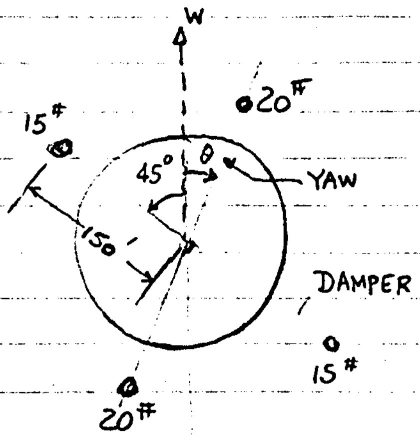
$$\left.\begin{aligned} dF_x &= 2\omega_0^2 x\,dm\left[8.5 + \left(\lambda - \frac{z}{x}\right)\sin 2\beta - \cos 2\beta\right] \\ dF_y &= -8\omega_0^2(x^2+z^2)^{1/2}\sin\left(\beta+\tan^{-1}\frac{x}{z}\right) dm \\ dF_z &= 2\omega_0^2 z\,dm\left[8.5 - \left(\lambda + \frac{x}{z}\right)\sin 2\beta + \cos 2\beta\right] \end{aligned}\right\} \quad (4)$$

(*) Noting that $\sin\left(\beta+\tan^{-1}\frac{x}{z}\right) = \sin\beta\,\frac{1}{\sqrt{1+\left(\frac{x}{z}\right)^2}} + \cos\beta\,\frac{\frac{x}{z}}{\sqrt{1+\left(\frac{x}{z}\right)^2}}$, this Equation can be written as follows:

$$dF_{y4} = -2\omega\omega_0(z\sin\beta + x\cos\beta)\,dm$$

TUMBLING PROBLEM.

SATELLITE CONFIGURATION – SYMMETRICAL



Determination of angle θ.

Product of inertia of four concentrated masses about x-z and y-z coordinate planes must be zero.

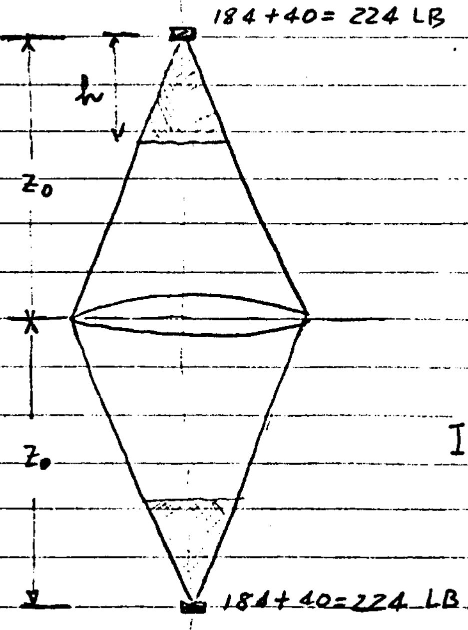
$$-(2)15(150\cos 45)(150\sin 45)+2(20)(150\cos\theta)(150\sin\theta)$$

or $\sin 2\theta = 0.75$, $2\theta = 48.6^{o}$, $\boxed{\theta = 24.3^{o}}$

Rod own weight : About 5.0 Lb/rod

YAW & DAMPER RODS & CONCENTRATED WEIGHTS:

$2\times 15+2\times 20+2\times 5 = 30+40+10 = 80$ LB (Although this weight goes 100% on the top, only half is considered there and the rest half @ the lower canister(*)



Lens, Rim

$$I_{ROLL} = 1,112,879 + 899,342 + (224\,z_0^2) + 154\times z_0^2$$

Sails

$$+2(11)\left[\frac{29.889 z_0}{18} + \frac{89221}{z_0} + \left\{z_0 - 3.6447\sqrt{z_0}\right\}^2\right]$$

$$+ 2\times 15\left[\left(\frac{150}{\sqrt{2}}\right)^2 + z_0^2\right] + 2\times 20\left[(150\times 0.91140)^2 + z_0^2\right]$$

$$= 3,097305 + 470.0\,z_0^2 + 328.775\,z_0 + \frac{1,962,862}{z_0} - 160.367\,z_0\sqrt{z_0}$$

$$I_{YAW} = 1,868,896 + 1,798,696 + 2\times 11\times\frac{89221}{z_0} + (2\times 15+2\times 20)\times 150^2$$

$$= 5,242,592 + \frac{1,962,862}{z_0}$$

(*) This can be done by redistribution of other weights (inflation system etc) in order to maintain the symmetry.

For $\frac{I_{ROLL}}{I_{YAW}} = 6^{(*)}$ we get

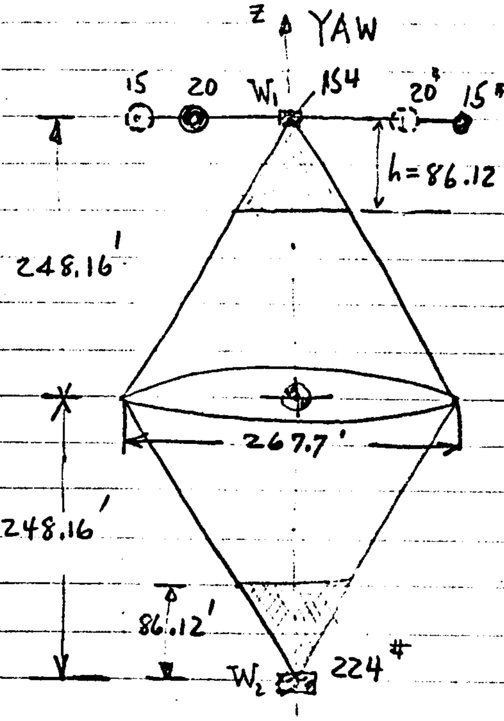
$$470\, z_0^2 + 328,775\, z_0 - 160,367\, z_0\sqrt{z_0} - \frac{9,814,310}{z_0} - 28,358,247 = 0.$$

or

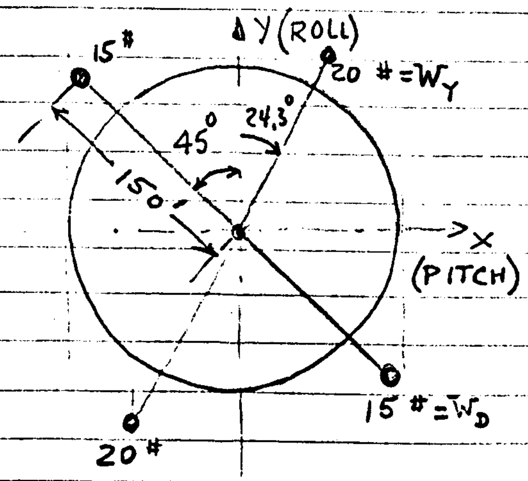
$$z_0^2 + 0.69952\, z_0 - 0.34121\, z_0\sqrt{z_0} - \frac{20881.5}{z_0} - 60336.7 = 0.$$

By trial & error $z_0 = 248.16$ Ft

For 4000 Ft^2/Sail, $h = 5.467\sqrt{z_0} = 86.12$ Ft



		Dimensions	Upper Mass	Lower Mass	Yaw Control Mass	Damper Control Mass
Notation			W_1	W_2	W_Y	W_D
Magnitude		LB	154	224	20	15
Coordinates	x	Ft	0	0	61.73	106.07
	y	Ft	0	0	136.71	−106.07
	z	Ft	248.16	−248.16	248.16	248.16



(*) In a second and better approximation the determination of the moments of inertia I_{ROLL} & I_{YAW} would include the four booms, which, as can be seen by their size and weight, would contribute an appreciable amount to the principal moments of inertia of the system.

TUMBLING ABOUT PITCH AXIS. $\left(\lambda = \frac{I_y - I_z}{I_y} = \frac{6-1}{6} = \frac{5}{6} = 0{,}8333 \right)$.

LOAD COMPONENTS ON,

UPPER MASS

$F_x = 0$

$F_y = -0.6943 \times 10^{-3} \sin 2\alpha$

$F_z = +0{,}6943 \times 10^{-3} \left[6{,}3333 - 0{,}8333 \sin 2\alpha + \cos 2\alpha\right]$

LOWER MASS,

$F_x = 0$

$F_y = +1{,}0100 \times 10^{-3} \sin 2\alpha$

$F_z = -1{,}0100 \times 10^{-3} \left[6{,}3333 - 0{,}8333 \sin 2\alpha + \cos 2\alpha\right]$

YAW MASS

$F_x = -0.01495 \times 10^{-3}$ lb

$F_y = +0.04967 \times 10^{-3} \left[6{,}3333 - 0{,}9819 \sin 2\alpha - \cos 2\alpha\right]$

$F_z = +0.09016 \times 10^{-3} \left[6{,}3333 - 1{,}3842 \sin 2\alpha + \cos 2\alpha\right]$

DAMPER MASS

$F_x = -0{,}01927 \times 10^{-3}$ lb

$F_y = -0.02890 \times 10^{-3} \left[6{,}3333 + 3{,}1729 \sin 2\alpha - \cos 2\alpha\right]$

$F_z = +0{,}06762 \times 10^{-3} \left[6{,}3333 - 0{,}4059 \sin 2\alpha + \cos 2\alpha\right]$

TUMBLING ABOUT ROLL AXIS.

LOAD COMPONENTS ON,

UPPER MASS

$F_x = -0{,}9256 \times 10^{-3} \sin 2\beta$

$F_y = -3.7025 \times 10^{-3} \sin \beta$

$F_z = +0{,}9256 \times 10^{-3} (8{,}5 - 0{,}8333 \sin 2\beta + \cos 2\beta)$

LOWER MASS

$F_x = +1{,}3463 \times 10^{-3} \sin 2\beta$

$F_y = +5{,}3853 \times 10^{-3} \sin \beta$

$F_z = -1{,}3463 \times 10^{-3} (8{,}5 - 0{,}8333 \sin 2\beta + \cos 2\beta)$

YAW MASS

$F_x = +0{,}0299 \times 10^{-3} (8{,}5 - 3.1867 \sin 2\beta - \cos 2\beta)$

$F_y = -0{,}4955 \times 10^{-3} \sin (\beta + 13^\circ 58')$

$F_z = +0{,}1202 \times 10^{-3} (8{,}5 - 1.0821 \sin 2\beta + \cos 2\beta)$

DAMPER MASS

$F_x = +0{,}0385 \times 10^{-3} (8{,}5 - 1.5063 \sin 2\beta - \cos 2\beta)$

$F_y = -0{,}3923 \times 10^{-3} \sin (\beta + 23^\circ 09')$

$F_z = +0{,}0902 \times 10^{-3} (8{,}5 - 1{,}2607 \sin 2\beta + \cos 2\beta)$

PLOT THESE FORCES FOR SEVERAL VALUES OF THE ANGLES α & β.

TABLE 1: LOAD COMPONENTS ON CONCENTRATED MASSES DUE TO GRAVITY GRADIENT AND INERTIA FORCES AND SATELLITE TUMBLING ABOUT ROLL AXIS.

ANGLE β [DEG]	F	UPPER MASS LB×10³	LOWER MASS LB×10³	YAW MASS LB×10³	DAMPER MASS LB×10³	ANGLE β [DEG]	F	UPPER MASS LB×10³	LOWER MASS LB×10³	YAW MASS LB×10³	DAMPER MASS LB×10³
0	x	0	0	0,2243	+0,2888	210	x	-0,8016	+1.1659	0,1567	+0,2578
	y	0	0	-0.1196	-0,1542		y	+1.8513	-2,6927	-0,3440	+0,3139
	z	8,7932	-12,790	+1,1419	+0,8569		z	7,6624	-11.145	+0.9692	+0,7133
30	x	-0,8016	+1.1659	0,1567	+0,2578	240	x	-0,8016	+1.1659	0.1866	+0,2963
	y	-1.8513	+2,6927	-0,3440	-0,3139		y	+3,2065	-4,6639	-0,4762	+0,3895
	z	7.6624	-11.145	+0,9692	+0,7133		z	6,7368	-9,799	+0,8490	+0,6231
60	x	-0,8016	+1,1659	0.1866	+0,2963	270	x	0	0	0,2841	+0.3658
	y	-3,2065	+4,6639	-0,4762	-0,3895		y	+3,7025	-5,3853	-0.4809	+0,3607
	z	6,7368	-9,799	+0,8490	+0,6231		z	6,9420	-10,097	+0,9015	+0,6765
90	x	0	0	0,2841	+0,3658	300	x	+0,8016	-1.1659	0,3516	+0,3967
	y	-3,7025	+5,3853	-0.4809	-0,3607		y	+3,2065	-4,6639	-0,3566	+0,2353
	z	6,9420	-10,097	+0,9015	+0,6765		z	8,0728	-11,742	+1,0742	+0,8201
120	x	+0,8016	-1,1659	0,3516	+0,3967	330	x	+0,8016	-1,1659	0,3218	+0,3582
	y	-3,2065	+4,6639	-0,3566	-0,2353		y	+1,8513	-2,6927	-0,1369	+0,0468
	z	8,0728	-11,742	+1,0742	+0,8201		z	8,9984	-13,088	+1,1944	+0,9103
150	x	+0,8016	-1,1659	0,3218	+0,3582	360	x	0	0	0,2243	+0,2888
	y	-1.8513	+2,6927	-0,1369	-0,0468		y	0	0	-0,1196	-0,1542
	z	8,9984	-13,088	+1,1944	+0,9103		z	8,7932	-12,790	+1,1419	+0,8569
180	x	0	0	0,2243	+0,2888		x				
	y	0	0	-0,1196	+0,1542		y				
	z	8,7932	-12,790	+1,1419	+0,8569		z				

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TUMBLING ABOUT ROLL AXIS

For convenience rewrite Equations 3:

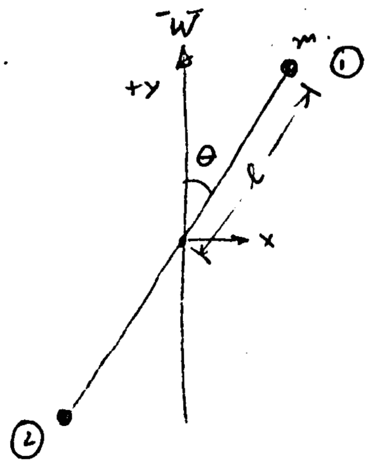
GENERAL FORCE EQUATIONS

$$\frac{dF_x}{dm} = \omega^2 x + 2\lambda x \omega_0^2 \sin 2\beta - 2z\omega_0^2 \sin 2\beta + x\omega_0^2 - 2x\omega_0^2 \cos 2\beta.$$

$$\frac{dF_y}{dm} = -2\omega\omega_0 \left(x\cos\beta + z\sin\beta\right)$$

$$\frac{dF_z}{dm} = \omega^2 z - 2\lambda z\omega_0^2 \sin 2\beta - 2x\omega_0^2 \sin 2\beta + z\omega_0^2 + 2z\omega_0^2 \cos 2\beta.$$

Forces & moments at the tetrapod apex due to concentrated forces at the Yaw masses



POINT	x	y	z
	COORDINATES		
1	$l\sin\theta$	$l\cos\theta$	h
2	$-l\sin\theta$	$-l\cos\theta$	h

$$\frac{1}{m}F_x = \omega^2 l\sin\theta + 2\lambda l\sin\theta\,\omega_0^2 \sin 2\beta - 2h\omega_0^2 \sin 2\beta + l\sin\theta\,\omega_0^2 - 2l\sin\theta\,\omega_0^2\cos 2\beta$$
$$- \omega^2 l\sin\theta - 2\lambda l\sin\theta\,\omega_0^2 \sin 2\beta - 2h\omega_0^2 \sin 2\beta - l\sin\theta\,\omega_0^2 + 2l\sin\theta\,\omega_0^2\cos 2\beta$$

$$= -4h\omega_0^2 \sin 2\beta$$

$$\frac{1}{m}F_y = -2\omega\omega_0\left(l\sin\theta\cos\beta + h\sin\beta\right) - 2\omega\omega_0\left(-l\sin\theta\cos\beta + h\sin\beta\right)$$

$$= -4\omega\omega_0 h\sin\beta.$$

$$\frac{1}{m}F_z = 2\omega^2 h - 4\lambda h\omega_0^2 \sin 2\beta + 2h\omega_0^2 + 4h\omega_0^2\cos 2\beta.$$

$$= 2h\left(\omega^2 + \omega_0^2\right) - 4h\omega_0^2\left(\lambda\sin 2\beta - \cos 2\beta\right).$$

$$\frac{1}{m} M_x = F_{z_1} \ell \cos\theta - F_{z_2} \ell \cos\theta$$

$$= 2\ell \cos\theta \left(- 2\ell \sin\theta \, \omega_0^2 \sin 2\beta \right)$$

$$= - 2\ell^2 \omega_0^2 \sin 2\theta \sin 2\beta \, .$$

$$\frac{1}{m} M_y = - F_{z1} \cdot \ell \sin\theta + F_{z2} \ell \sin\theta$$

$$= - 2\ell \sin\theta \left(- 2\ell \sin\theta \, \omega_0^2 \sin 2\beta \right)$$

$$= + 4 \ell^2 \omega_0^2 \sin^2\theta \sin 2\beta .$$

$$\frac{1}{m} M_z = - F_{x1} \ell \cos\theta + F_{y1} \ell \sin\theta + F_{x2} \ell \cos\theta - F_{y2} \ell \cos\theta$$

$$= - \ell \cos\theta \left(F_{x1} - F_{x2} \right) + \ell \sin\theta \left(F_{y1} - F_{y2} \right)$$

$$= - \ell^2 \sin 2\theta \left[\omega^2 + \omega_0^2 + 2\omega_0^2 \left(\lambda \sin 2\beta - \cos 2\beta \right) \right]$$

$$- 4 \omega \omega_0 \ell^2 \sin^2\theta \cos\beta .$$

SUMMARY

$$\frac{1}{m} F_x = - 4 h \omega_0^2 \sin 2\beta$$

$$\frac{1}{m} F_y = - 4 h \omega \omega_0 \sin\beta$$

$$\frac{1}{m} F_z = 2h \left[(\omega^2 + \omega_0^2) - 2\omega_0^2 \left(\lambda \sin 2\beta - \cos 2\beta \right) \right]$$

$$\frac{1}{m} M_x = - 2\ell^2 \omega_0^2 \sin 2\theta \sin 2\beta$$

$$\frac{1}{m} M_y = + 4\ell^2 \omega_0^2 \sin^2\theta \sin 2\beta$$

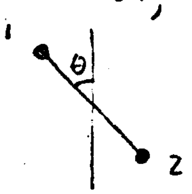
$$\frac{1}{m} M_z = - \ell^2 \sin 2\theta \left[(\omega^2 + \omega_0^2) + 2\omega_0^2 \left(\lambda \sin 2\beta - \cos 2\beta \right) \right] - 4 \omega \omega_0 \ell^2 \sin^2\theta \cos\beta .$$

<u>Forces & Moments @ the Tetrapod Apex Due to Concentrated masses at the Damper Masses.</u>

Let ϕ be the angle the damper rod makes with the rim plane

(Angle of rod – polar axis $= \frac{\pi}{2} - \phi$)

Then,



Point	Coordinates: x	y	z
1	$\ell\cos\phi\sin\theta$	$\ell\cos\phi\cos\theta$	$h+\ell\sin\phi$
2	$-\ell\cos\phi\sin\theta$	$-\ell\cos\phi\cos\theta$	$h-\ell\sin\phi$

$$\frac{1}{m}F_x = -2(h+\ell\sin\phi)\omega_0^2\sin2\beta - 2(h-\ell\sin\phi)\omega_0^2\sin2\beta$$
$$= -4h\omega_0^2\sin2\beta.$$

$$\frac{1}{m}F_y = -2\omega\omega_0(h+\ell\sin\phi)\sin\beta - 2\omega\omega_0(h-\ell\sin\phi)\sin\beta$$
$$= -4\omega\omega_0 h\sin\beta$$

$$\frac{1}{m}F_z = \Big\{\omega^2(h+\ell\sin\phi) - 2\lambda\omega_0^2(h+\ell\sin\phi)\sin2\beta + \omega_0^2(h+\ell\sin\phi) + 2\omega_0^2(h+\ell\sin\phi)\cos2\beta\Big\} + \Big\{\omega^2(h-\ell\sin\phi) - 2\lambda\omega_0^2(h-\ell\sin\phi)\sin2\beta + \omega_0^2(h-\ell\sin\phi) - 2\omega_0^2(h-\ell\sin\phi)\cos2\beta\Big\}$$
$$= 2\Big\{\omega^2 h - 2\lambda\omega_0^2 h\sin2\beta + \omega_0^2 h + 2\omega_0^2 h\cos2\beta\Big\}$$
$$= 2h\Big[(\omega^2+\omega_0^2) - 2\omega_0^2(\lambda\sin2\beta - \cos2\beta)\Big]$$

$$\frac{1}{m}M_x = F_{z1}\,\ell\cos\phi\cos\theta - F_{z2}\,\ell\cos\phi\cos\theta =$$
$$\ell\cos\phi\cos\theta\Big[\omega^2(2\ell\sin\phi) - 2\lambda\omega_0^2(2\ell\sin\phi)\sin2\beta - 4\omega_0^2\ell\cos\phi\sin\theta\sin2\beta + \omega_0^2\cdot2\ell\sin\phi + 2\omega_0^2\cdot2\ell\sin\phi\cos2\beta\Big]$$
$$= \ell^2\cos\theta\sin2\phi\Big[(\omega^2+\omega_0^2) - 2\omega_0^2(\lambda\sin2\beta - \cos2\beta)\Big] - 2\ell^2\omega_0^2\cos^2\phi\sin2\theta\sin2\beta$$

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$$\frac{1}{m} M_y = - F_{z1}\, l \cos\phi \sin\theta + F_{z2}\, l \cos\phi \sin\theta$$

$$= - l \cos\phi \sin\theta \left[2l \sin\phi\right]\left[\omega^2 + \omega_0^2 - 2\omega_0^2 \left(\lambda \sin 2\beta - \cos 2\beta\right)\right]$$

$$- l \cos\phi \sin\theta \left(- 4\omega_0^2 l \cos\phi \sin\theta \sin 2\beta\right)$$

$$= -2l^2 \sin\theta \sin 2\phi \left[\omega^2 + \omega_0^2 - 2\omega_0^2 \left(\lambda \sin 2\beta - \cos\beta\right)\right] + 4 l^2 \omega_0^2 \sin^2\theta \cos^2\phi \sin 2\beta$$

$$\frac{1}{m} M_z = - l \cos\phi \cos\theta \left(F_{x1} - F_{x2}\right) + l \cos\phi \sin\theta \left(F_{y1} - F_{y2}\right)$$

$$= -2l \cos\phi \cos\theta \Big[\underline{\omega^2 l \cos\phi \sin\theta} + 2\lambda \omega_0^2 l \cos\theta \sin\theta \sin 2\beta - 2\omega_0^2 l \sin\phi \sin 2\beta$$

$$+ \underline{\omega_0^2 l \cos\phi \sin\theta} - 2\omega_0^2 l \cos\phi \sin\theta \cos 2\beta \Big]$$

$$+ l \cos\phi \sin\theta \left(-2\omega\omega_0\right)\left(2l \cos\phi \sin\theta \cos\beta + 2l \sin\phi \sin\beta\right)$$

$$= - l^2 \cos^2\phi \sin 2\theta \left[\omega^2 + \omega_0^2 + 2\omega_0^2 \left(\lambda \sin 2\beta - \cos 2\beta\right)\right] + 2 l^2 \omega_0^2 \cos\theta \sin 2\phi \sin 2\beta$$

$$- 4 l^2 \omega\omega_0 \cos\phi \sin\theta \left(\cos\phi \sin\theta \cos\beta + \sin\phi \sin\beta\right).$$

SUMMARY.

$$\frac{1}{m} F_x = -4h\omega_0^2 \sin 2\beta$$

$$\frac{1}{m} F_y = -4\omega\omega_0 h \sin\beta$$

$$\frac{1}{m} F_z = 2h\left[\left(\omega^2 + \omega_0^2\right) - 2\omega_0^2\left(\lambda \sin 2\beta - \cos 2\beta\right)\right]$$

$$\frac{1}{m} M_x = l^2 \cos\theta \sin 2\phi \left[\omega^2 + \omega_0^2 - 2\omega_0^2 \left(\lambda \sin 2\beta - \cos 2\beta\right)\right] - 2 l^2 \omega_0^2 \cos^2\phi \sin 2\theta \sin 2\beta$$

$$\frac{1}{m} M_y = -2l^2 \sin\theta \sin 2\phi \left[\omega^2 + \omega_0^2 - 2\omega_0^2 \left(\lambda \sin 2\beta - \cos 2\beta\right)\right] + 4 l^2 \omega_0^2 \sin^2\theta \cos^2\phi \sin 2\beta$$

$$\frac{1}{m} M_z = \begin{cases} - l^2 \cos^2\phi \sin 2\theta \left[\omega^2 + \omega_0^2 + 2\omega_0^2 \left(\lambda \sin 2\beta - \cos 2\beta\right)\right] + 2 l^2 \omega_0^2 \cos\theta \sin 2\phi \sin 2\beta \\ - 4 l^2 \omega\omega_0 \cos\phi \sin\theta \left(\cos\phi \sin\theta \cos\beta + \sin\phi \sin\beta\right) \end{cases}$$

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TABLE 2:

FORCE AND MOMENT COMPONENTS AT THE TETRAPOD APEX CAUSED BY FORCES AT THE CONCENTRATED MASSES OF THE,

YAW ROD	DAMPER ROD
$\frac{1}{m_Y} F_x = -4h\omega_o^2 \sin 2\beta$ $\frac{1}{m_Y} F_y = -4h\omega\omega_o \sin\beta$ $\frac{1}{m_Y} F_z = 2h\left[\omega^2+\omega_o^2-2\omega_o^2(\lambda\sin 2\beta-\cos 2\beta)\right]$ $\frac{1}{m_Y} M_x = -2\ell^2\omega_o^2 \sin 2\theta \sin 2\beta$ $\frac{1}{m_Y} M_y = +4\ell^2\omega_o^2 \sin^2\theta \sin 2\beta$ $\frac{1}{m_Y} M_z = -\ell^2 \sin 2\theta\left[\omega^2+\omega_o^2+2\omega_o^2(\lambda\sin 2\beta-\cos 2\beta)\right]$ $\quad -4\omega\omega_o\ell^2\sin^2\theta\cos\beta$.	$\frac{1}{m_D} F_x = -4h\omega_o^2 \sin 2\beta$ $\frac{1}{m_D} F_y = -4h\omega\omega_o \sin\beta$ $\frac{1}{m_D} F_z = 2h\left[\omega^2+\omega_o^2-2\omega_o^2(\lambda\sin 2\beta-\cos 2\beta)\right]$ $\frac{1}{m_D} M_x = \ell^2\cos\theta\sin 2\phi\left[\omega^2+\omega_o^2-2\omega_o^2(\lambda\sin 2\beta-\cos 2\beta)\right]$ $\quad -2\ell^2\omega_o^2\cos^2\phi\sin 2\theta\sin 2\beta$ $\frac{1}{m_D} M_y = -2\ell^2\sin\theta\sin 2\phi\left[\omega^2+\omega_o^2-2\omega_o^2(\lambda\sin 2\beta-\cos 2\beta)\right]$ $\quad +4\ell^2\omega_o^2\sin^2\theta\cos^2\phi\sin 2\beta$. $\frac{1}{m_D} M_z = -\ell^2\cos^2\phi\sin 2\theta\left[\omega^2+\omega_o^2+2\omega_o^2(\lambda\sin 2\beta-\cos 2\beta)\right]$ $\quad +2\ell^2\omega_o^2\cos\theta\sin 2\phi\sin 2\beta$ $\quad -4\ell^2\omega\omega_o\cos\phi\sin\theta(\cos\phi\sin\theta\cos\beta-\sin\phi\sin\beta)$
NUMERICAL VALUES: $\omega_o^2 = 0.39\times 10^{-6}\ \sec^{-1}$ $\omega = 4\omega_o$, $\ell = 150$ Ft, $h = 248.16$ Ft, $\theta = 24.3^\circ$, $m_Y = 20/32.2 = 0.6211$ Slug, $\lambda = \frac{5}{6}$	$\omega_o, \omega, \ell, h, \lambda$, as in yaw rod. $\theta = -45^\circ$, $m_D = 15/32.2 = 0.4658$ slug.
$10^3 F_x = -0.2404\sin 2\beta$ lb $10^3 F_y = -0.9618\sin\beta$ lb $10^3 F_z = +0.2404\left(8.5-\frac{5}{6}\sin 2\beta+\cos 2\beta\right)$ lb $M_x = -0.0981\sin 2\beta$ in-lb $M_y = +0.0443\sin 2\beta$ in-lb $M_z = -0.0981\left(8.5+\frac{5}{6}\sin 2\beta-\cos 2\beta+1.8060\cos\beta\right)$ in-lb	$10^3 F_x = -0.1803\sin 2\beta$ lb $10^3 F_y = -0.7212\sin 2\beta$ lb $10^3 F_z = +0.1803\left(8.5-\frac{5}{6}\sin 2\beta+\cos 2\beta\right)$ lb $M_x = 0.0694\left[\left(8.5-\frac{5}{6}\sin 2\beta+\cos 2\beta\right)\sin 2\phi\right.$ $\quad \left.+1.4142\cos^2\phi\sin 2\beta\right]$ in-lb $M_y = 0.1387\left[\left(8.5-\frac{5}{6}\sin 2\beta+\cos 2\beta\right)\sin 2\phi\right.$ $\quad \left.+0.7071\cos^2\phi\sin 2\beta\right]$ in-lb. $M_z = +0.0981\left[\left(8.5+\frac{5}{6}\sin 2\beta-\cos 2\beta\right)\cos^2\phi\right.$ $\quad +0.7071\sin 2\phi\sin 2\beta$ $\quad \left.-5.6569\cos\phi(0.7071\cos\phi\cos\beta+\sin\phi\sin\beta)\right]$ in-lb.

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SUPERPOSITION OF YAW ROD & DAMPER ROD LOADS FOR $\phi = 0°$

$10^3 F_x = -0.4207 \sin 2\beta$

$10^3 F_y = -1.6830 \sin 2\beta.$

$10^3 F_z = +0.4207 \left(8.5 - \frac{5}{6} \sin 2\beta + \cos 2\beta\right)$

$M_x = 0$

$M_y = +0.1424 \sin 2\beta$

$M_z = -0.5696 \cos \beta$

TABLE 3: FORCES AND MOMENTS AT THE TETRAPOD APEX DUE TO CONCETRATED MASSES AT THE TIPS OF YAW & DAMPER RODS WHEN $\phi = 0°$.

β	0	20	40	60	80	100	120	140	160	180
$10^3 F_x$	0	-.2704	-.4143	-.3643	-.1439	+.1439	+.3643	+.4143	+.2704	0
$10^3 F_y$	0	-1,0818	-1,6574	-1.4575	-.5756	+.5756	+1.4575	+1.6574	+1.0818	0
3,5760	3,5760	3,5760	3,5760	3,5760	3,5760	3,5760	3,5760	3,5760	3,5760	3,5760
$-.3506 \sin 2\beta$	0	-.2254	-.3453	-.3036	-.1199	+.1199	+.3036	+.3453	+.2254	0
$+.4207 \cos 2\beta$	+.4207	+.3223	+.0731	-.2104	-.3953	-.3953	-.2104	+.0731	+.3223	+.4207
$10^3 F_z = \Sigma \rightarrow$	+3.9967	+3.6729	+3.3038	+3.0620	+3.0608	+3.3006	+3.6692	+3.9944	+4.1237	3.9967
M_x	0	0	0	0	0	0	0	0	0	0
M_y	0	+.0915	+.1402	+.1233	+.0487	-.0487	-.1233	-.1402	-.0915	0
M_z	-.5696	-.5352	-.4363	-.2848	-.0989	+.0989	+.2848	+.4363	+.5352	+.5646

β	200	220	240	260	280	300	320	340	360
$10^3 F_x$	-.2704	-.4143	-.3643	-.1439	+.1439	+.3643	+.4143	+.2704	0
$10^3 F_y$	-1,0818	-1.6574	-1.4575	-.5756	+.5756	+1.4575	+1.6574	+1.0818	
3,5760→	3.5760	3,5760	3.5760	3.5760	3.5760	3.5760	3.5760	3.5760	3.5760
$-.3506 \sin 2\beta$	-.2254	-.3453	-.3036	-.1199	+.1199	+.3036	+.3453	+.2254	0
$+.4207 \cos 2\beta$	+.3223	+.0731	-.2104	-.3953	-.3953	-.2104	+.0731	+.3223	+.4207
$10^3 F_z = \Sigma \rightarrow$	+3.6729	3,3038	3.0620	3.0608	3,3006	3.6692	3.9944	4.1237	3.9967
M_x	0	0	0	0	0	0	0	0	0
M_y	+.0915	+.1402	+.1233	+.0487	-.0487	-.1233	-.1402	-.0915	0
M_z	+.5352	+.4363	+.2848	+.0989	-.0989	-.2848	-.4363	-.5352	-.5696

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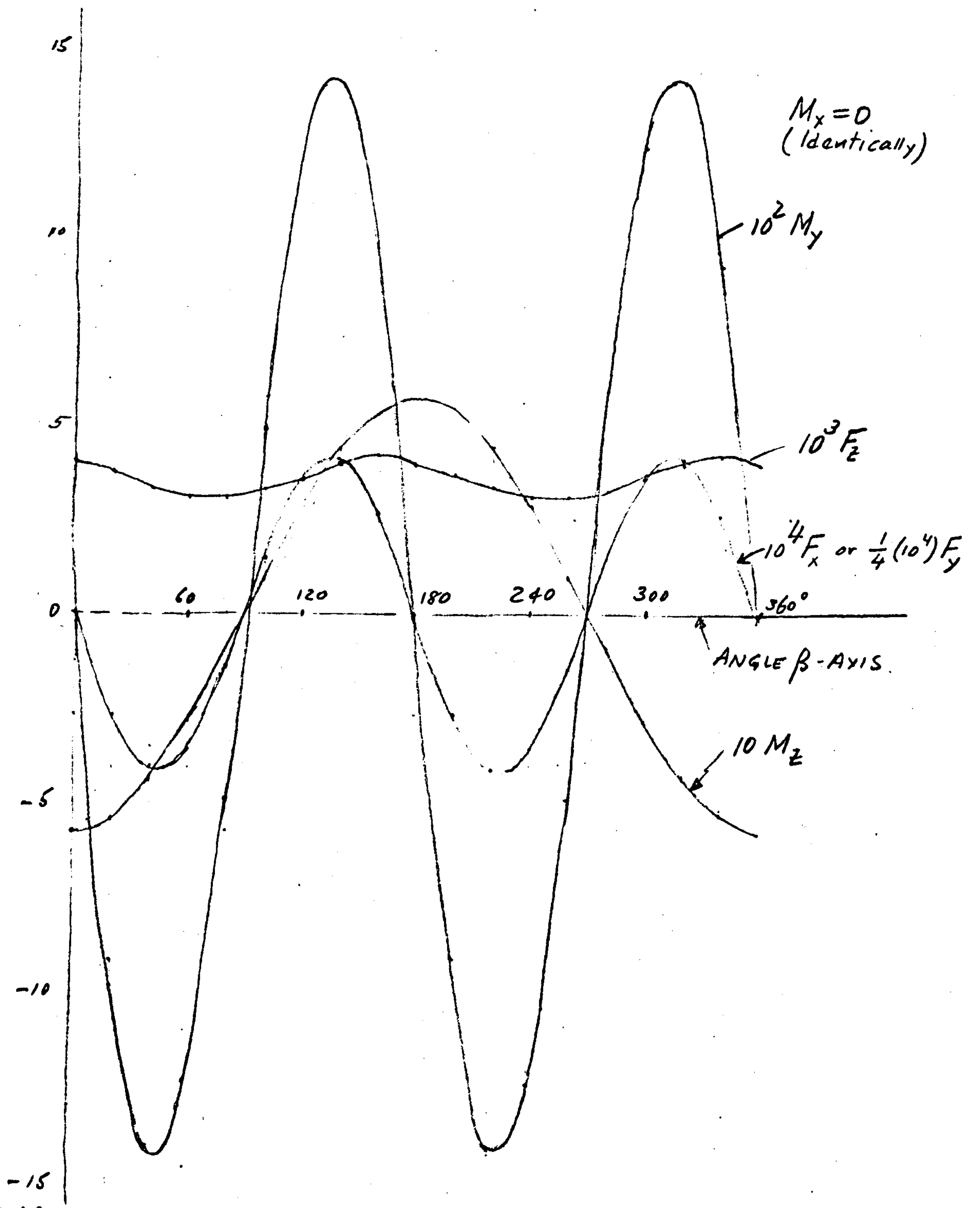

FIGIRE 1:

FORCES (LB) & MOMENTS (in-lb) AT THE TETRAPOD APEX CAUSED BY THE FOUR MASSES AT THE ENDS OF THE YAW & DAMPER ROD ($\varphi = 0°$) FOR A SYMMETRICAL SATELLITE TUMBLING ABOUT THE ROLL AXIS

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SUPERPOSITION OF YAW ROD & DAMPER ROD LOADS FOR $\phi = 45°$

$$\left.\begin{aligned} 10^3 F_x &= -0,4207 \sin 2\beta \\ 10^3 F_y &= -1,6830 \sin 2\beta \\ 10^3 F_z &= +0,4207\left(8,5 - \tfrac{5}{6}\sin 2\beta + \cos 2\beta\right) \end{aligned}\right\} \text{SAME AS } \phi = 0°$$

$$M_x = +0.0694\,(8,5 - 1.5404 \sin 2\beta + \cos 2\beta)$$

$$M_y = +0,1387\,(8,5 - 0,1600 \sin 2\beta + \cos 2\beta)$$

$$M_z = -0,0491\,(8,5 - 0,5809 \sin 2\beta - \cos 2\beta) - 0,3734 \cos\beta - 0,2775 \sin\beta.$$

TABLE 4: FORCES AND MOMENTS AT THE TETRAPOD APEX DUE TO CONCENTRATED MASSES AT THE TIPS OF YAW & DAMPER RODS WHEN $\phi = 45°$

	β	0	20	40	60	80	100	120	140	160	180
1	8,5000	8,5000	8,5000	8,5000	8,5000	8,5000	8,5000	8,5000	8,5000	8,5000	8,5000
2	$\cos 2\beta$	1.0000	0,7660	0.1737	-0,5000	-0,9397	-0,9397	-0.5000	+0.1737	+0.7660	+1.0000
3	1 + 2	9.5000	9,2660	8.6737	8,0000	7,5603	7,5603	8.000	8.6737	9.2660	9.5000
4	$-1,5404 \sin 2\beta$	0	-0,9902	-1,5170	-1,3340	-0,5268	+0,5268	+1.3340	+1,5170	+0,9902	0
5	$-0,16 \sin 2\beta$	0	-0.1028	-0.1576	-0.1386	-0.0547	+0,0547	+0.1386	+0.1576	+0.1028	0
6	$-0,5809 \sin 2\beta$	0	-0,3734	-0,5721	-0,5031	-0.1987	+0,1987	+0,5031	+0,5721	+0,3734	0
7	3 + 4	9.5000	+8,2758	7.1567	6.6660	7.0335	8.0871	9.3340	10.1907	10.2562	9.5000
→ 8	$M_x = 0.0694 \times$ 7	0.6593	0,5743	0.4967	0.4626	0.4881	0.5612	0.6478	0,7072	0,7118	0.6593
9	3 + 5	9.5000	9,1632	8,5161	7.8614	7,5056	7.6150	8.1386	8,8313	9,3688	9,5000
→ 10	$M_y = 0.1387 \times$ 9	1,3177	1,2709	1,1812	1.0904	1.0410	1.0562	1.1288	1,2249	1.2995	1.3177
11	1 − 2 + 6	7,5000	7,3606	7.7542	8.4969	9,2410	9,6384	9,5031	8.8984	8.1074	7,5000
12	$-0.0491 \times$ 11	-0.3683	-0,3614	-0,3807	-0,4172	-0.4537	-0.4732	-0.4666	-0.4369	-0,3981	-0,3683
13	$-0,3734 \cos\beta$	-0,3734	-0,3509	-0,2860	-0.1867	-0,0648	+0.0648	+0,1867	+0,2860	+0.3509	+0,3734
14	$-0,2775 \sin\beta$	0	-0,0949	-0.1784	-0,2403	-0,2733	-0,2733	-0.2403	-0,1784	-0.0949	0
→ 15	$M_z =$ 12 + 13 + 14	-0,7417	-0,8072	-0,8451	-0,8442	-0,7918	-0.6817	-0.5202	-0,3293	-0,1421	+0.0051

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LEGEND:
FORCES F_x, F_y & F_z SAME AS IN CASE $\phi = 0$.

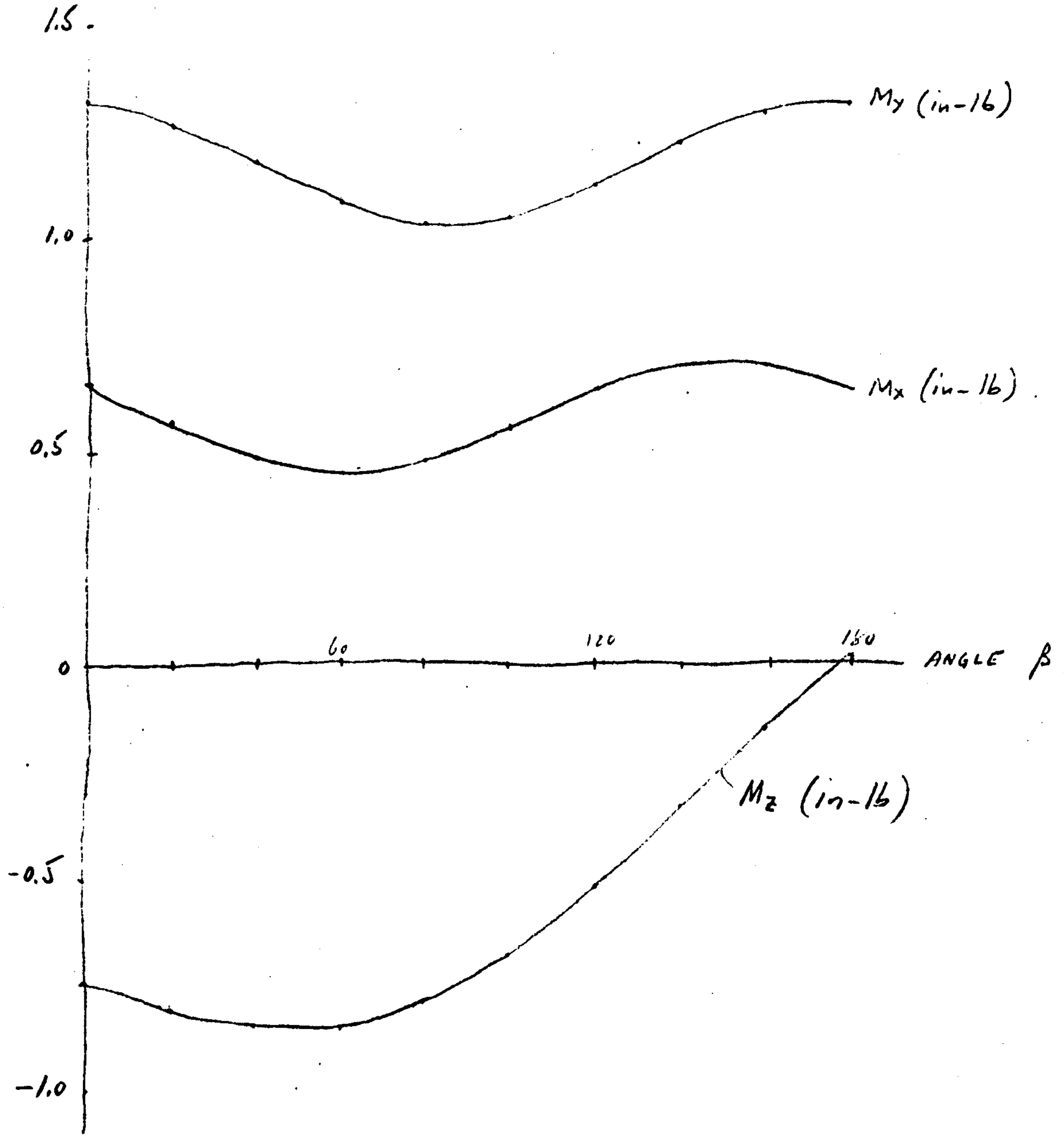

FIG. 2:
MOMENTS (in-lb) AT THE TETRAPOD APEX CAUSED BY THE FOUR MASSES AT THE ENDS OF THE YAW & DAMPER ROD ($\phi = 45°$) FOR A SYMMETRICAL SATELLITE TUMBLING ABOUT THE ROLL AXIS.

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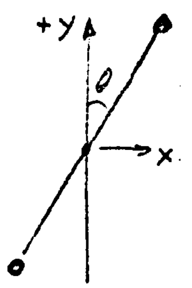
MAXIMUM TIP DEFLECTION OF YAW ROD.



Force components at the end of the yaw rod. [See Eqs (3) of page 4]

where,

$$x = l \sin\theta = 0.4115\, l \quad Ft.$$
$$z = h = 248.16 \quad Ft.$$
$$dm = \frac{20}{32.2} = 0.6211 \quad Slug.$$

$$F_x = 0.6211\, \omega_0^2 \left[\left(\frac{\omega}{\omega_0}\right)^2 (0.41115 l) + 2(0.4115 l)\left(\tfrac{5}{6}\right)\sin 2\beta - 0.4115 l\,(1 - 4\sin^2\beta) - 4\,(248.16)\sin\beta\cos\beta\right]$$

OR

$$F_x = 0.1994\, l\, 10^{-6}\left[\frac{1}{2}\left(\frac{\omega}{\omega_0}\right)^2 + \frac{1}{2} + \frac{5}{6}\sin 2\beta - \cos 2\beta - \frac{603.06}{l}\sin 2\beta\right]$$

$$F_y = -0.1994\, l\, 10^{-6}\left(\frac{\omega}{\omega_0}\right)\cdot\left[\frac{603.06}{l}\sin\beta + \cos\beta\right] \qquad (5)$$

$$F_z = 0.1994\, l\, 10^{-6}\left[\frac{301.53}{l}\left\{1 + \left(\frac{\omega}{\omega_0}\right)^2\right\} - \frac{603.06}{l}\left(\frac{5}{6}\sin 2\beta - \cos 2\beta\right) - \sin 2\beta\right]$$

For $l = 150$ Ft & $\omega = 4\omega_0$ the above equations become

$$F_x = +0.0299 \times 10^{-3}\,(8.5 - 3.1867\sin 2\beta - \cos 2\beta)$$
$$F_y = -0.1196 \times 10^{-3}\,(4.0204\sin\beta + \cos\beta) \qquad (6)$$
$$F_z = +0.1202 \times 10^{-3}\,(8.5 - 1.0821\sin 2\beta + \cos 2\beta)$$

(See also Page 7)

This force as a vector $\bar{F}$ is

$$\bar{F} = F_x\,\bar{i}' + F_y\,\bar{j}' + F_z\,\bar{k}' \qquad (7)$$

($\bar{i}', \bar{j}'$ & $\bar{k}'$ are unit vectors along the x, y & z axes).

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Coordinates of yaw semi-rod:

$$l_x = l \sin\theta = 150\,(0.4115) = 61.73 \text{ Ft}$$

$$l_y = l \cos\theta = 150\,(0.9114) = 136.71 \text{ Ft}$$

$$l_z = 0.$$

and $$\bar{l} = l_x \bar{i} + l_y \bar{j} + \cancel{l_z \bar{k}} \qquad (8)$$

Unit vector along $\bar{l}$ is

$$\bar{l_o} = \frac{l_x}{l}\bar{i}' + \frac{l_y}{l}\bar{j}' + \frac{l_z}{l}\bar{k}' \qquad (l = 150 \text{ Ft})$$

$$= 0.4115\,\bar{i}' + 1,9114\,\bar{j}' + 0. \qquad (9)$$

The force component, F_n, normal to the yaw rod at its tip is, the magnitude of the cross product $\bar{F} \times \bar{l_o}$, which is,

$$\sqrt{\begin{vmatrix} F_y & F_z \\ l_{oy} & l_{oz} \end{vmatrix}^2 + \begin{vmatrix} F_z & F_x \\ l_{oz} & l_{ox} \end{vmatrix}^2 + \begin{vmatrix} F_x & F_y \\ l_{ox} & l_{oy} \end{vmatrix}^2}$$

$$= \sqrt{(F_x^2 + F_y^2 + F_z^2)(l_{ox}^2 + l_{oy}^2 + l_{oz}^2) - (F_x l_{ox} + F_y l_{oy} + F_z l_{oz})^2}$$

$$= \sqrt{(F_x^2 + F_y^2 + F_z^2) - (F_x l_{ox} + F_y l_{oy})^2}$$

or

$$F_n^2 = F_x^2 + F_y^2 + F_z^2 - (0.4115 F_x + 0.9114 F_y)^2$$

$$= 0.9114 F_x^2 + 0.4115 F_y^2 + F_z^2 - 0.75 F_x F_y.$$

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or,

$$10^6 F_n^2 = 0.9114(0.000894)(8.5 - 3.1867 \sin 2\beta - \cos 2\beta)^2$$
$$+ 0.4115(0.014304)(4.0204 \sin\beta + \cos\beta)^2$$
$$+ 0.014448(8.5 - 1.0821 \sin 2\beta + \cos 2\beta)^2$$
$$+ 0.002682(8.5 - 3.1867 \sin 2\beta - \cos 2\beta)(4.0204 \sin\beta + \cos\beta)$$

Simplifying the above equation yields

$$10^6 F_n^2 = 1.17348 + 0.09277 \sin\beta + 0.00293 \cos\beta$$
$$- 0.28626 \sin 2\beta - 0.27639 \cos 2\beta$$
$$- 0.00966 \sin 3\beta + 0.01450 \cos 3\beta$$
$$- 0.01304 \sin 4\beta - 0.00496 \cos 4\beta . \qquad [lb^2]$$

TABLE 5: NORMAL FORCE AT THE TIP OF THE YAW ROD FOR VARIOUS β's.

$\beta \rightarrow$	0°	20°	40°	60°	80°	100°	120°	140°	160°	180°
1.17348 →	1.17348	1.17348	1.17348	1.17348	1.17348	1.17348	1.17348	1.17348	1.17348	1.17348
+0.09277 sin β	0	.03173	+.05963	+.08034	+.09136	+.09136	+.08034	+.05963	+.03173	0
+0.00293 cos β	+.00293	.00275	+.00224	+.00147	+.00051	-.00051	-.00147	-.00224	-.00275	-.00293
-.28626 sin 2β	0	-.18401	-.28191	-.24791	-.09791	+.09791	+.24791	+.28191	+.18401	0
-.27639 cos 2β	-.27639	-.21173	-.04800	+.13820	+.25972	+.25972	+.13820	-.04800	-.21173	-.27639
-.00966 sin 3β	0	-.00837	-.00837	0	+.00837	+.00837	0	-.00837	-.00837	0
+.01450 cos 3β	+.01450	+.00725	-.00725	-.01450	-.00725	+.00725	+.01450	+.00725	-.00725	-.01450
-.01304 sin 4β	0	-.01284	-.00446	+.01129	+.00838	-.00838	-.01129	+.00446	+.01284	0
-.00496 cos 4β	-.00496	-.00086	+.00466	+.00248	-.00380	-.00380	+.00248	+.00466	-.00086	-.00496
Σ →	+.90956	+.79740	+.89002	+1.14485	+1.43286	+1.62540	+1.64415	+1.47278	1.17110	+.87470

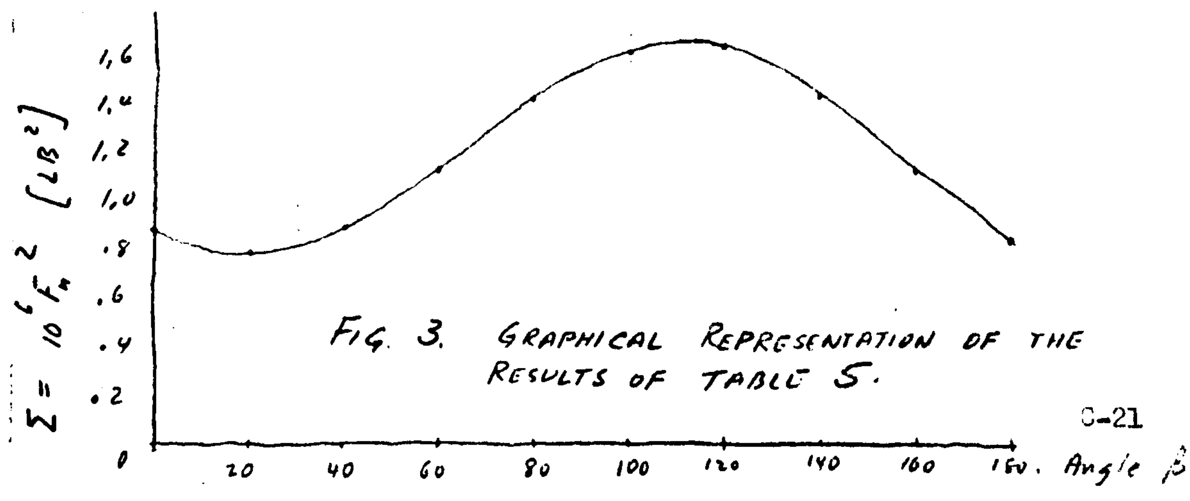


FIG. 3. GRAPHICAL REPRESENTATION OF THE RESULTS OF TABLE 5.

JDM

Nov. 24/64

The axial force in the yaw rod is the dot product $\bar{F} \cdot \bar{l}_0$

Hence,

$$10^3 F_a = F_x l_{ox} + F_y l_{oy} + F_z l_{oz}$$

$$= 0.0299 (0.4115) (8.5 - 3.1867 \sin 2\beta - \cos 2\beta)$$

$$- 0.1196 (.9114) (4.0204 \sin \beta + \cos \beta)$$

$$= 0.10458 - 0.03921 \sin 2\beta - 0.01230 \cos 2\beta - 0.43822 \sin \beta - 0.10900 \cos \beta$$

TABLE 6: YAW ROD AXIAL FORCE (NEG. SIGN MEANS COMPRESSION)

$\beta \rightarrow$	0°	20°	40°	60°	80°	100°	120°	140°	160°	180°
0.10458	0.10458	0.10458	0.10458	0.10458	0.10458	0.10458	0.10458	0.10458	0.10458	0.10458
$-.43822 \sin\beta$	0	-.14988	-.28168	-.37951	-.43156	-.43156	-.37951	-.28168	-.14988	0
$-.10900 \cos\beta$	-.10900	-.10243	-.08350	-.05450	-.01893	+.01893	+.05450	+.08350	+.10243	+.10900
$-.03921 \sin 2\beta$	0	-.02520	-.04861	-.03396	-.02520	+.01341	+.03396	+.04861	+.02520	0
$-.01230 \cos 2\beta$	-.01230	-.00942	-.00214	+.00615	+.01156	+.01156	+.00615	-.00214	-.00942	-.01230
$\Sigma \rightarrow$	-.01672	-.18235	-.31135	-.35724	-.35955	-.28308	-.18032	-.04713	+.07291	+.20128

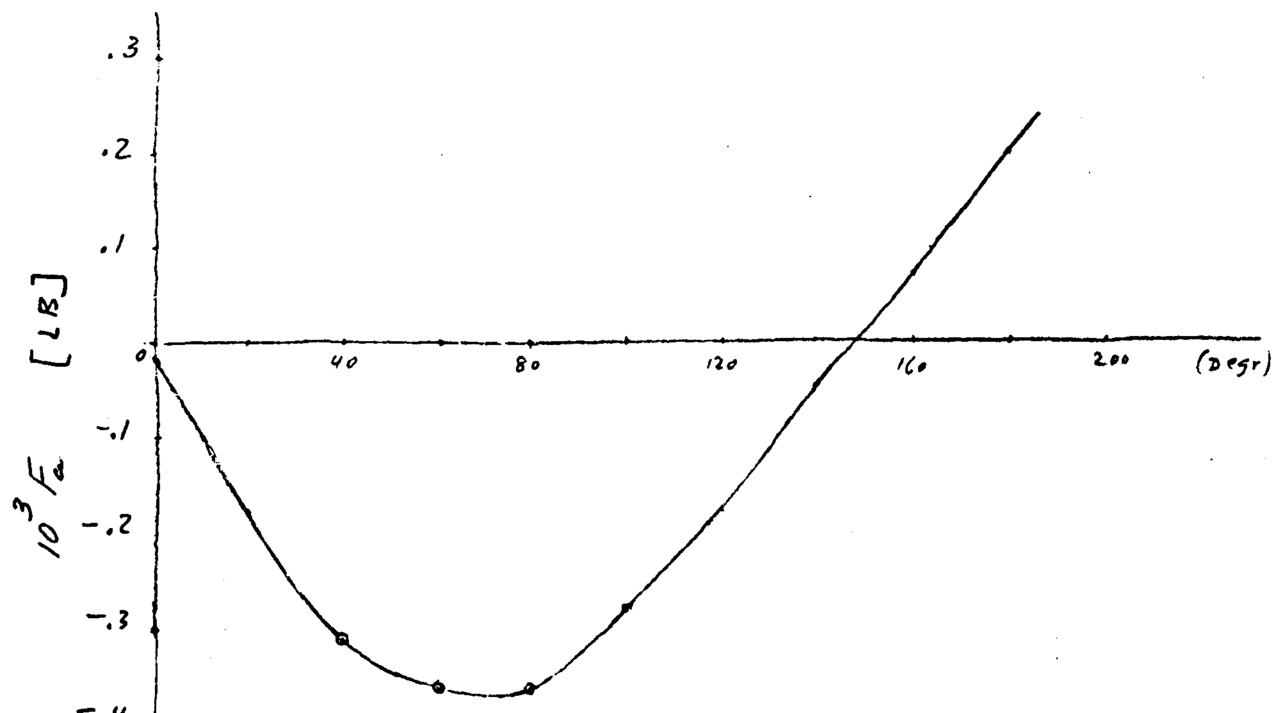
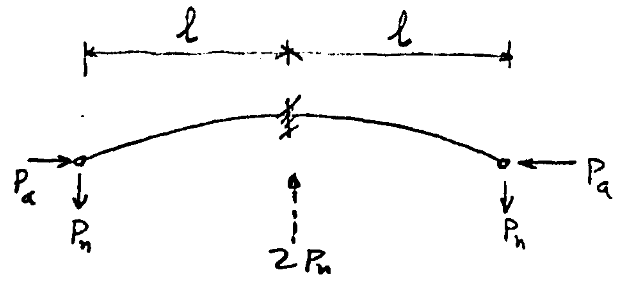


FIG. 4: GRAPHICAL REPRESENTATION OF THE RESULTS OF TABLE 6.

JDM

NOV. 25/64.



$l = 150$ ft $= 1800''$

Use De Havilland tube $1\frac{1}{8}''$ DIA.
Beryllium copper
$E = 19\times10^6$ psi
$EI = 72{,}000$ lb-in^2.

$$P_{EULER} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(72{,}000)}{(2\times1800)^2} = \frac{0{,}71064}{12{,}96} = 0.055 \text{ lb.}$$

P_{ACTUAL} (max) $\cong 0.36\times10^{-3}$ (@ $\beta = 80°$ - See Page 17)

$$\frac{P}{P_{EULER}} = \frac{0{,}36}{55} = 0.0065 \text{ (Negligible.)}$$

Deflection due to transverse load

$$\delta_b = \frac{P_n l^3}{3EI} = \frac{\sqrt{1{,}7\times10^{-6}}\ (1800)^3}{3\times72{,}000} = \frac{1{,}3\times10^{-3}\times5.832\times10^9}{.216\times10^6} = 35.1''$$

Estimated thermal deflection $\delta_T \cong 40''$

Total deflection, $\delta = \delta_b + \delta_T \cong 75''$.

DEC. 1, 64.

SIZE AND MAXIMUM DEFLECTION OF TETRAPOD BOOMS

For convenience the maximum forces and moments at the tetrapod apex due to tip loads at the yaw and damper rods are summarized in the following table.

TABLE 7:(*) MAX. FORES & MOMENTS AT APEX DUE TO TIP LOADS AT YAW & DAMPER ROD

	F_x [lb]	F_y [lb]	F_z [lb]	M_x [in-lb]	M_y [in-lb]	M_z [in-lb]
$\phi = 0$	4.2×10^{-4} ($\beta=135°$)	16.8×10^{-4} ($\beta=135°$)	4.1×10^{-3} ($\beta=160°$)	0	14.2×10^{-2} ($\beta=135°$)	0.57 ($\beta=180°$)
$\phi = 45°$	SAME ↑	SAME ↑	SAME ↑	0.72 ($\beta=150°$)	1.32 ($\beta=180°$)	0.85 ($\beta=50°$)

Critical is the condition $\phi = 45°$.

For a limit angle of twist $\theta = 5°$ (for the booms) due to $M_z = 0.85$ in-lb the total weight of all four booms is about 75 lb. (Reference 2)

Using the notation of Reference 3, page J-3 the radius r_o of the boom can be found from equation

$$r_o = \frac{s}{\pi d}\sqrt[3]{\frac{5\pi \ell M_z}{12 d E_w \theta \cos\alpha_u}}$$

(*) These values of F & M and the corresponding values of the angle β were taken from Figures 1 & 2.

Letting $d = 0.005''$ (wire diameter)
$s = 0.125$ (spacing of axial wires)
$E_w = 10^7$ psi (wire modulus of elasticity)
$\theta = 5^\circ = 0.08727$ Radians
$M_z = 0.85$ in-lb
$\ell = (248.16^2 + 133.85^2)^{1/2} = 281.96$ Ft $= 3384$ in
$\cos\alpha_n = h/\ell = 0.88012$

the above equation yields

$$r_0 = 7.9''$$

Then boom diameter $D = 15.8''$.
(Half mil photolyzable film is used in making the booms. Film density $= 0.038$ lb/in^3).

For convenience we rewrite the force components on the upper mass itself (See page 7)

$$\left.\begin{aligned} F_x &= -0.9256 \times 10^{-3} \sin 2\beta \\ F_y &= -3.7025 \times 10^{-3} \sin\beta \\ F_z &= 0.9256 \times 10^{-3}(8.5 - 0.8333\sin 2\beta + \cos 2\beta). \end{aligned}\right\} \quad (10)$$

For the booms critical is the angle $\beta = 70^\circ$ which produces a minimum tension in the booms due to F_z and a relatively large compression due to F_y.

Suppose that the F_y (@ $\beta = 70^\circ$) of Equations (10) is directly additive to the critical F_y (@ $\beta = 135^\circ$) of Table 7 then $F_{y\,max} = \left[3.7025 \sin 70^\circ + 1.68\right] \times 10^{-3} = 5.16 \times 10^{-3}$ lb.

From the 3^d of Eqs (10), for $\beta = 70°$

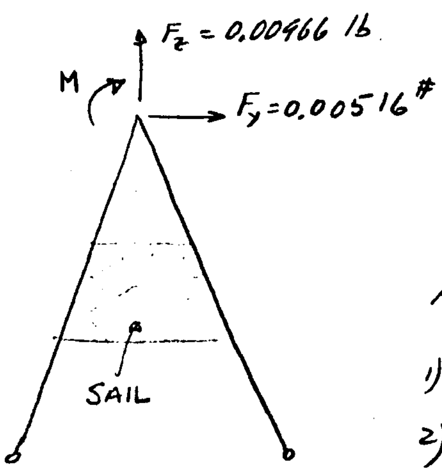
$$F_z = 0.9256\times10^{-3}(8.5-0.5357-0.7660) = 6.663\times10^{-3}\ lb.$$

From Figure 1 the minimum F_z is about 3×10^{-3} lb.

Hence, total minimum $F_z^{(*)}$ is

$$F_z = 9.663\times10^{-3}\ lb$$

Tension per leg of tetrapod : $\dfrac{9.663\times10^{-3}}{4\cos\alpha_u} = 0.00274$ lb.



Take $M = 1.32$ in-lb$^{(**)}$ (The moment M_y of condition $\phi = 45$ at $\beta = 180°$ – See Table 7)

Axial load in boom:

1) Due to F_z = +0.00274 LB.

2) Due to M ; $\dfrac{M\ell}{2Rh}$ =

$$-\frac{1.32\,(281.96)}{2(133.85)(248.16)(12)} = -0.00047 \quad "$$

3) Due to F_y : $-\dfrac{0.00516}{\dfrac{2\times133.85}{281.96}} = -0.00544$ "

Total compressive load: -0.00317 LB.

Assume a solar radiation pressure of 10^{-9} lb/in^2 on sail.
Sail area = 8000 Ft^2

Total solar pressure $= 8000\times144\times10^{-9} = 1.152\times10^{-3}$ lb.

Load per boom $= \frac{1}{2}(1.152)\times10^{-3} = 0.576\times10^{-3}$ lb.

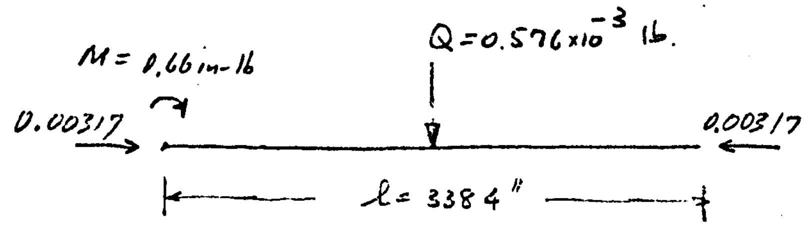
Assume that this load is concentrated at the midpoint of the boom (Load Q of sketch-page 26).

(*) The value of $F_z = 3\times10^{-3}$ lb from the rod tip masses occurs at about the same angle $\beta = 70°$

(**) This is a conservative condition since the max M that goes with F_y of the above sketch is 0.72 in-lb according to Table 7

$$M_{max} = \frac{1}{2}(0.66) + \frac{0.576\times10^{-3}\times3384}{4}$$

$$= 0.33 + 0.487 = 0.817 \text{ in-lb}$$



BOOM CRITICALLY LOADED.

The moment M=0.66 in-lb at the left end of the boom in the above sketch is half the moment $M_y = 1.32$ in-lb (See page 25)

From Reference 3, page G-2,

$$P_{cr} = \frac{1.21\times10^8 r_0^3 d}{K\ell^2} = \frac{1.21\times10^8(7.9)^3(0.005)}{25\times(3384)^2} = 1.04 \text{ lb} \gg 0.00317 \text{ lb}$$

$$M_{cr} = \frac{7.5 r_0 \times 10^6 d^2}{K^{3/2}} = \frac{7.5\times7.9\times10^6\times25\times10^{-6}}{125} = 11.85 \text{ in-lb} \gg 0.817 \text{ in-lb}$$

Moment of inertia, I, of boom cross section:

$$I = \frac{\pi D^2 d^2 n_a}{32} \quad \text{(See Ref. 3 - Page V-6 Eq. 24)}$$

or $$I = \frac{\pi(15.8)^2(25)10^{-6}\,\pi(7.9)/0.125}{32} = 0.12166 \text{ in}^4$$

Deflection at midpoint due to Q:

$$\delta_1 = \frac{Q\ell^3}{48EI} = \frac{0.576\times10^{-3}(3384)^3}{48(\frac{1}{2}\times10^7)\,0.12166} = 0.67'' \quad (E = \tfrac{1}{2}\times10^7 \text{ psi})$$

Maximum deflection (not at midpoint) due to end moment M=0.66 in-lb

$$\delta_2 = \frac{(0.0616)(0.66)(3384)^2}{\frac{1}{2}10^7(0.12166)} = 0.80''$$

Total deflection $\delta_0 < \delta_1 + \delta_2 = 0.67 + 0.80 = 1.47''$

Correction according to equation $\delta = \delta_0 \div (1 - P/P_{cr})$ is not necessary because the ratio P/P_{cr} is negligibly small.

J.D. Marketos 7/4/56

Appendix D

GOODYEAR AEROSPACE CORPORATION

ENGINEERING MEMORANDUM REPORT

December 16, 1964
SM-8834

Subject: Canister Separation Velocity Study

References:

1. Mandel, J. A., Compression Buckling Tests of Wire Film Cylinders, GER 11771, Goodyear Aerospace Corporation, October 1964.

2. Packaging Sequence, SP-2768, Goodyear Aerospace Corporation, January 4, 1964.

3. Marketos, J. D., Tumbling Satellite, Goodyear Aerospace Corporation Engineering Memorandum Report SM-8827, December 1, 1964.

4. Marketos, J. S., Asymmetric Lensat Configuration with Ames Damper, etc., Goodyear Aerospace Corporation Engineering Memorandum Report SM-8828, December 4, 1964.

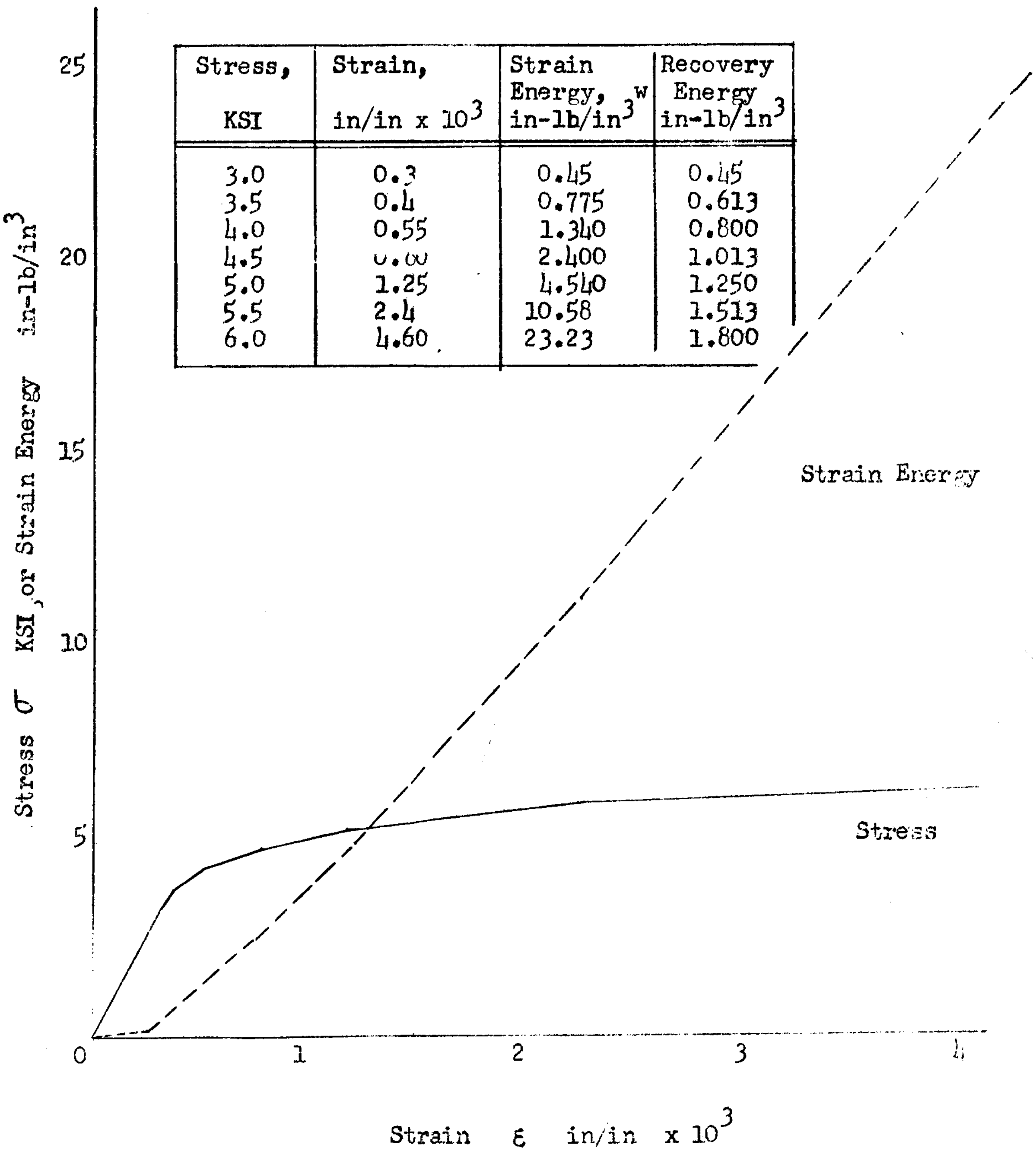
1. Symmetric Satellite Configuration

Assume that the separation of the two half canisters occurs at a rate of 3 Ft/Sec for each half (224 lb, Reference 3 page 5), relative to the lens-torus-rim assumed stationary. The kinetic energy of each half canister is

$$K.E = \frac{1}{2} M V^2 = \frac{1}{2} \frac{224}{32.2} 3^2 = 31.32 \text{ Ft-lb} = 375.8 \text{ in-lb.}$$

Figure 1 is a replot of Figure 9, page 59 of Reference 1 for aluminum wire (dia. = 5.0 mil). In the same figure, the strain energy per unit volume is plotted versus the strain, and in a table on page 2 corresponding values are listed of the stress, strain, strain energy per unit volume, and the recovery energy, i.e. the strain energy minus the dissipated energy. The axial wires in one boom (Reference 3, page 24) have a volume of

$$\frac{\pi(0.005)^2}{4} \times \frac{2\pi(7.9)}{0.125} \times 3384 \text{ or } 26.4 \text{ in}^3.$$



Stress, KSI	Strain, in/in x 10^3	Strain Energy, w in-lb/in^3	Recovery Energy in-lb/in^3
3.0	0.3	0.45	0.45
3.5	0.4	0.775	0.613
4.0	0.55	1.340	0.800
4.5	[illegible]	2.400	1.013
5.0	1.25	4.540	1.250
5.5	2.4	10.58	1.513
6.0	4.60	23.23	1.800

Figure 1: Stress and Strain Energy Per Unit Volume Versus Strain For 5.0 Mil Aluminum Wire

a. Suppose that the energy of 375.8 in-lb. is stored as strain energy in only one boom. This is conceivable because, in packaging the satellite one of the booms starts getting twisted as soon as the rim starts getting wound on the drum (Reference 2); therefore when the entire rim has been wound, this boom will be the shortest of all four in each tetrapod. When the rim is 50% wound two additional booms in each tetrapod start getting twisted, hence their final length in the packaged satellite will be somewhat larger than the first boom and shorter than the fourth boom which has not been twisted at all.

In the case where the one boom is effective, the energy of 375.8 in-lb would require a strain energy density of $375.8/26.4 = 14.23$ in-lb/in^3.

As can be seen from the table of page 2 or from Figure 2 the axial aluminum wires are stressed at a level a little higher than 5,500 psi, and most of the energy is dissipated as plastic flow. The remainder of the strain energy (recovery energy) is 1.560 in-lb/in^3 which according to Figure 2 would cause the half canisters to return towards each other at a relative velocity (with respect to the stationary lens) of about 1.15 Ft/sec. Considering the smallness of the K.E. the low return velocity of the half canisters, the fact that booms and torus would be at that time in the process of inflation, and that some energy has to be absorbed by the undoing of the lens, torus and booms folds, it is easy to visualize that the half-canisters will not return clear back to their original position. There will possibly be an oscillation about the final equilibrium position until the energy is entirely dissipated.

b. If three booms are effective the required strain energy density would be $375.8/(3 \times 26.4) = 4.75$ in-lb/in^3. This would cause an average of about 5000 psi stress in the axial wires of these booms and a recovery energy of about 1.30 in-lb/in^3 (see Figure 1, graph or table) and according to Figure 2 this energy would cause the canisters to return at a relative velocity (with respect to the stationary midpoint) of $0.52V_s$ Ft/sec or 1.56 Ft/sec.

Comparison of cases a. and b. shows that one boom effective is better than three booms effective, because the former leads to a smaller return velocity. In fact, with smaller return velocity, the inflation of the booms and torus would be more advanced and the chances for faster dissipation of the remaining energy appear to be better. The fact that the recovery energy in this case is a little larger than in the case of three effective booms does not seem to be as serious a problem as the return velocity.

Figure 2: Separation-to-Return Velocity Ratio Versus Stress Level In The Axial Aluminum Wires of the Tetrapod Booms of a Symmetric Satellite.

V_s = Separation Velocity

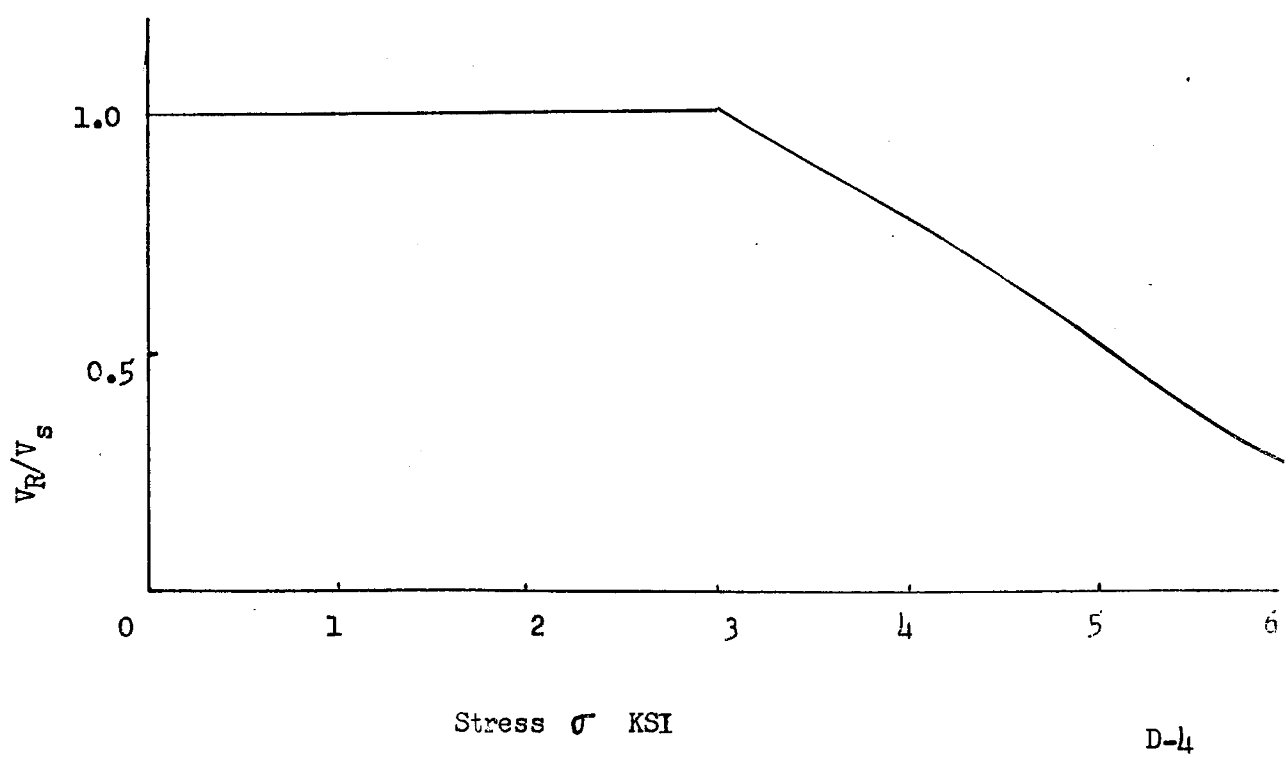
V_R = Return Velocity

W_s = Strain Energy (Total)

W_R = Recovery Energy

$$\frac{V_R}{V_S} = \sqrt{\frac{W_R}{W_S}}$$

σ	W_s	W_R	W_R/W_s	$\sqrt{\frac{W_R}{W_s}}$
3,000	0.45	0.45	1.000	1.000
3,500	0.775	0.613	0.791	0.889
4,000	1.340	0.800	0.597	0.773
4,500	2,400	1.013	0.422	0.650
5,000	4.540	1.250	0.275	0.524
5,500	10.58	1.513	0.143	0.378
6,000	23.23	1.800	0.0775	0.278



2. Asymmetric Satellite Configuration

Symbols:

M_c = Mass of heavy half-canister

M_1 = Mass of lens-rim-torus group

V_s = Common velocity (opposite signs) of two half canisters at the first instant of separation (Velocity of lens-rim-torus is zero)

V_F = Common velocity of the heavy half-canister and the attached to it satellite, as an increment or decrement of the orbital velocity

Equations:

Conservation of Momentum

$$V_F (M_c + M_1) = V_s M_c \quad \text{or} \quad V_F = \frac{V_s M_c}{M_c + M_1} \tag{1}$$

Initial kinetic energy: $\frac{1}{2} M_c V_s^2$ (2)

Final kinetic energy: $\frac{1}{2} (M_c + M_1) V_F^2$ (3)

Energy to be dissipated,

$$\Delta E = \frac{1}{2} M_c V_s^2 - \frac{1}{2} (M_c + M_1) V_F^2 =$$

$$\frac{1}{2} V_s^2 \left[M_c - \frac{M_c^2}{(M_c + M_1)^2} (M_c + M_1) \right] = \frac{1}{2} V_s^2 \cdot \frac{M_c M_1}{M_c + M_1}$$

Noting that $M_c = 413$ lb, $M_1 = 772$ (Reference 4), the above equation yields:

$$\Delta E = \frac{1}{2} \left(\frac{269}{g} \right) V_s^2 .$$

GOODYEAR AEROSPACE CORPORATION

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SM-8834

Comparison of this energy to be dissipated in the satellite, with the energy $\frac{1}{2} \frac{224}{g} V_s^2$ of the symmetrical configuration (see page 1) shows that the former is by about 20% larger than the latter, hence the energy dissipation problem here is for all practical purposes identical to that of the symmetrical configuration.

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GOODYEAR AEROSPACE CORPORATION

ENGINEERING MEMORANDUM REPORT

DEC. 14, 1964
SM 8835

SUBJECT: Symmetrical Satellite Configuration. Moments of Inertia about Principal Axes During Various Stages of Deployment.

References:

1. Asymmetric Lensat Configuration with Ames Damper. Tetrapod size etc... . Goodyear Aerospace Corporation Engineering Memorandum Report SM 8828, Dec. 4, 1964.

2. Feasibility Study and Preliminary Design of Gravity Gradient Stabilized Lenticular Test Satellite, Interim Technical Report, Contract NAS-1-3114 GER 11502, June 1964.

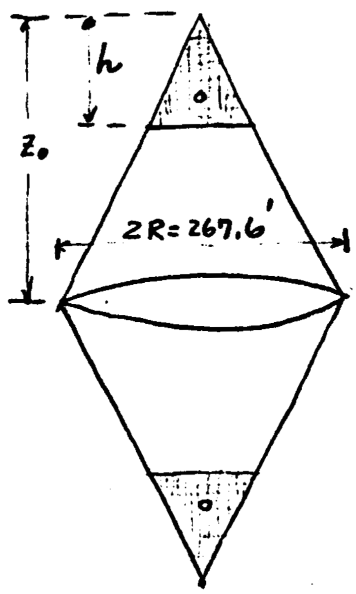
Symmetrical Satellite Configuration.

Equal weights at the tetrapod apeces : 184 lb.
Sail total area : 2 × 4000 = 8000 Ft^2
Sail total weight : 22 lb.
Yaw control : Two - 20 lb masses located on the rim.
(in the orbital plane under normal flight).
Coordinate axes :

x-x Roll axis (Tangent to orbit when satellite is in normal flight)
y-y Pitch axes
z-z Yaw axis (Local vertical under satellite normal flight).

Requirement: $I_{x-x}/I_{z-z} = 6$ (operational).

Assume a net weight (only wire material) of 75 lb for each tetrapod,



One sail area : $\frac{1}{2} h \frac{2Rh}{z_o} = \frac{Rh^2}{z_o} = 4000$

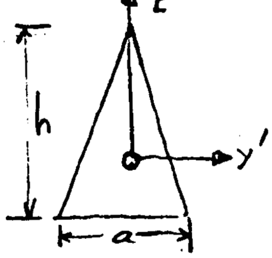
or $h = \sqrt{\frac{4000 z_o}{133.8}} = 5.467\sqrt{z_o}$.

Moments of inertia of triangular plate about centroidal axes : (See sketch on the right)

$$I_{y'} = \frac{Wh^2}{18}$$

$$I_z = \frac{Wa^2}{24}$$

$$I_{x'} = I_{y'} + I_z = \frac{W}{72}(4h^2 + 3a^2)$$



W = Plate weight (= 11.0 lb per triang. sail).

For moments of inertia of booms about centroidal axes see Reference 1, page 3.

Component	Weight [lb]	Distance from CG	I_{x-x} (ROLL) LB − FT^2	I_{z-z} (YAW) LB − FT^2
LENS	199.0	0	1,112,879	1,868,896
RIM	100.4	0	899,348	1,798,696
UPPER OR LOWER MASS	184.0	z_0	$184\, z_0^2$	0
" " " SAIL	11.0	$z_0 - 3.6447\sqrt{z_0}$	(1)	(2)
" " " TETRAPOD	75.0	$z_0/2$	(3)	(4)
YAW CONTROL	40.0	0	0	716,420

(1) : $11.0\left[\frac{h^2}{18} + \frac{1}{6}\left(\frac{Rh}{z_0}\right)^2 + \left(z_0 - \frac{2}{3}h\right)^2\right]$

(2) : $11.0\left(\frac{1}{6}\right)\left(\frac{Rh}{z_0}\right)^2$

(3) $\frac{75}{12}\left(2R^2 + z_0^2\right) + 75\frac{z_0^2}{4} = \frac{75}{6}\left(R^2 + 2z_0^2\right) = 223{,}780 + 25 z_0^2$.

(4) $\frac{75}{3}R^2 = 25R^2 = 447{,}561$ FT^2-LB.

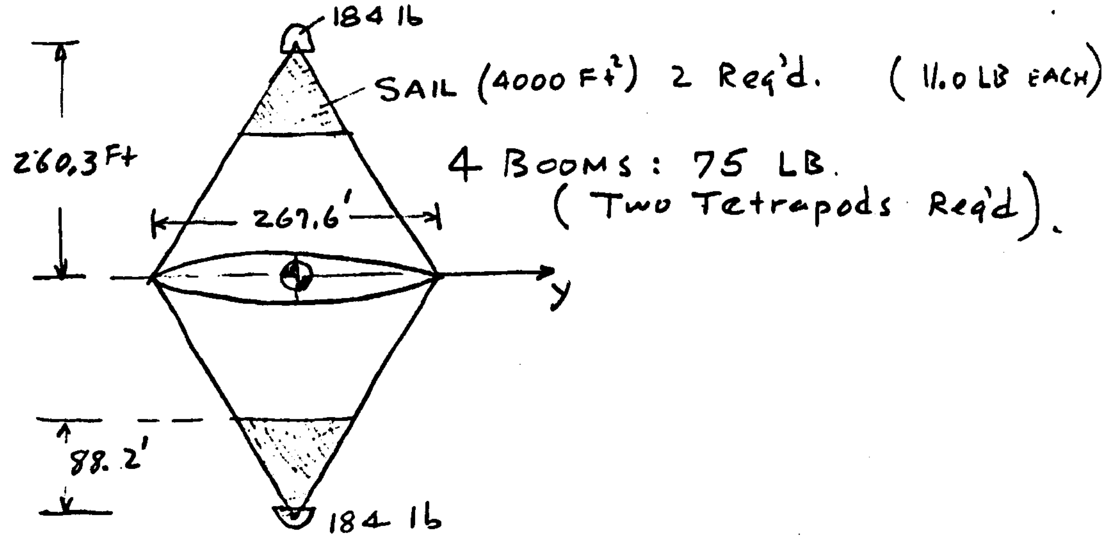
Then equation $I_{x-x} = 6\, I_{z-z}$ leads to

$$1{,}112{,}879 + 899{,}348 + 368\, z_0^2 + 22\left[1.6604\, z_0 + \frac{89178.3}{z_0} + \left(z_0 - 3.4647\sqrt{z_0}\right)^2\right]$$
$$+ 447561 + 50\, z_0^2 = 6\left[1{,}868{,}896 + 1{,}798{,}696 + 22\left\{\frac{89178.3}{z_0}\right\}\right.$$
$$\left. + 2(447561) + 716{,}420\right]$$

or $z_0^2 + 0.6832\, z_0 - 0.3465\, z_0\sqrt{z_0} - \frac{22294.6}{z_0} - 66397.8 = 0$

from which (by trial and error) $z_0 = 260.3$ Ft.

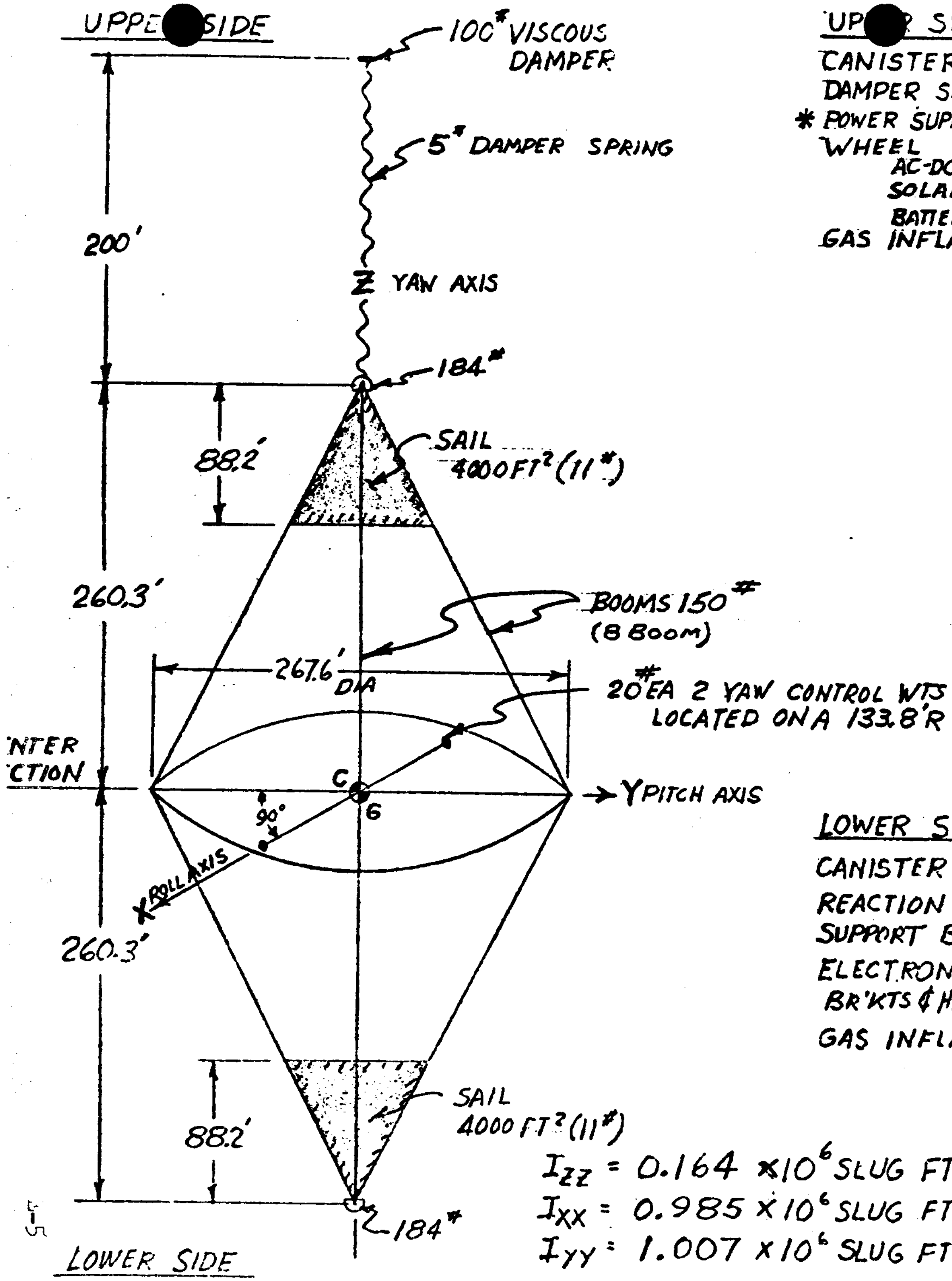
Weights and Moments of Inertia at the final stage of deployment



COMPONENT	WEIGHT [LBS]	DISTANCE FROM CG. [FT]	I_{x-x} (ROLL) LB-FT^2	I_{Y-Y} (PITCH) LB-FT^2	I_{z-z} (YAW) LB-FT^2
LENS	199.0	0	1 112 879	1,112,879	1,868,896
RIM	100.4	0	899 348	899 348	1,798 696
UPPER MASS	184.0	260.3	12,467,104	12,467,104	0
LOWER MASS	184.0	260.3	12,467,104	12,467,104	0
UPPER SAIL	11.0	204.4	468,096	464,373	3,924
LOWER SAIL	11.0	204.4	468,096	464,373	3,924
UPPER TETRAPOD	75.0	125.0	1,917,680	1,917,680	447,561
LOWER TETRAPOD	75.0	125.0	1,917,680	1,917,680	447,561
YAW CONTROL	40.0	0	0	716,420	716420
$\Sigma \longrightarrow$			31,717,987	32,426,961	5,286,982

$$\frac{I_{x-x}}{I_{z-z}} = 6.00 \quad ; \quad \frac{I_{Y-Y}}{I_{x-x}} = 1.022.$$

In page 5 the weight break down of the various components and the dimensions of the satellite are shown.



UP R SIDE WTS

CANISTER SHELL	40#
DAMPER SUPPORT	10#
* POWER SUPPLY FOR REACTION WHEEL	3#
AC-DC INVERTER 1#	
SOLAR CELLS 1#	
BATTERIES 1#	
GAS INFLATION SYSTEM	131#
TOTAL	184#

* NOTE
POWER REQUIREMENTS BASED ON USING 43 WATTS ON A 15 MIN OPERATION ONCE A WEEK

CENTER SECTION WTS

TORUS (PHOTOLYZABLE)	117#
RIM	103
LENS (PHOTOLYZABLE)	552#
FILM 353#	
WIRE 199#	
YAW CONTROL WT	40#
TOTAL	812#

LOWER SIDE WTS

CANISTER SHELL	40#
REACTION WHEEL	25#
SUPPORT BR'KT & HARDWARE	4#
ELECTRONIC CONTROLS	18#
BR'KTS & HARDWARE	2#
GAS INFLATION SYSTEM	95#
TOTAL	184#

LAUNCH WT

VISCOUS DAMPER	105#
UPPER SIDE	184#
CENTER SECTION	812#
LOWER SIDE	184#
SAIL	22#
GAS	17#
BOOMS	150#
SEPARATION SYS	10#
TOTAL	1484#
ORBITAL WT	967#

$I_{ZZ} = 0.164 \times 10^6$ SLUG FT²

$I_{XX} = 0.985 \times 10^6$ SLUG FT²

$I_{YY} = 1.007 \times 10^6$ SLUG FT²

$I_{XX}/I_{ZZ} = 6$

$I_{YY}/I_{XX} = 1.022$

SATELLITE OPERATIONAL SYMMETRICAL CONFIGURATION

H.W.B. 11-1-64
REV A 12-16-64

Appendix E

Satellite moments of inertia about principal axes, and expecially ratios of such moments of inertia are critical for the successful deployment of the satellite. To show how such quantities vary during various stages of deployment five key positions were selected (see table of page 8). The time given in each stage of deployment is after separation from the vehicle.

Figure 1 shows the canister 1 sec. after separation from the vehicle, but before the canister separation.

Figure 2 shows the satellite at the 100 sec. position with the canister halves fully extended, just before inflation starts.

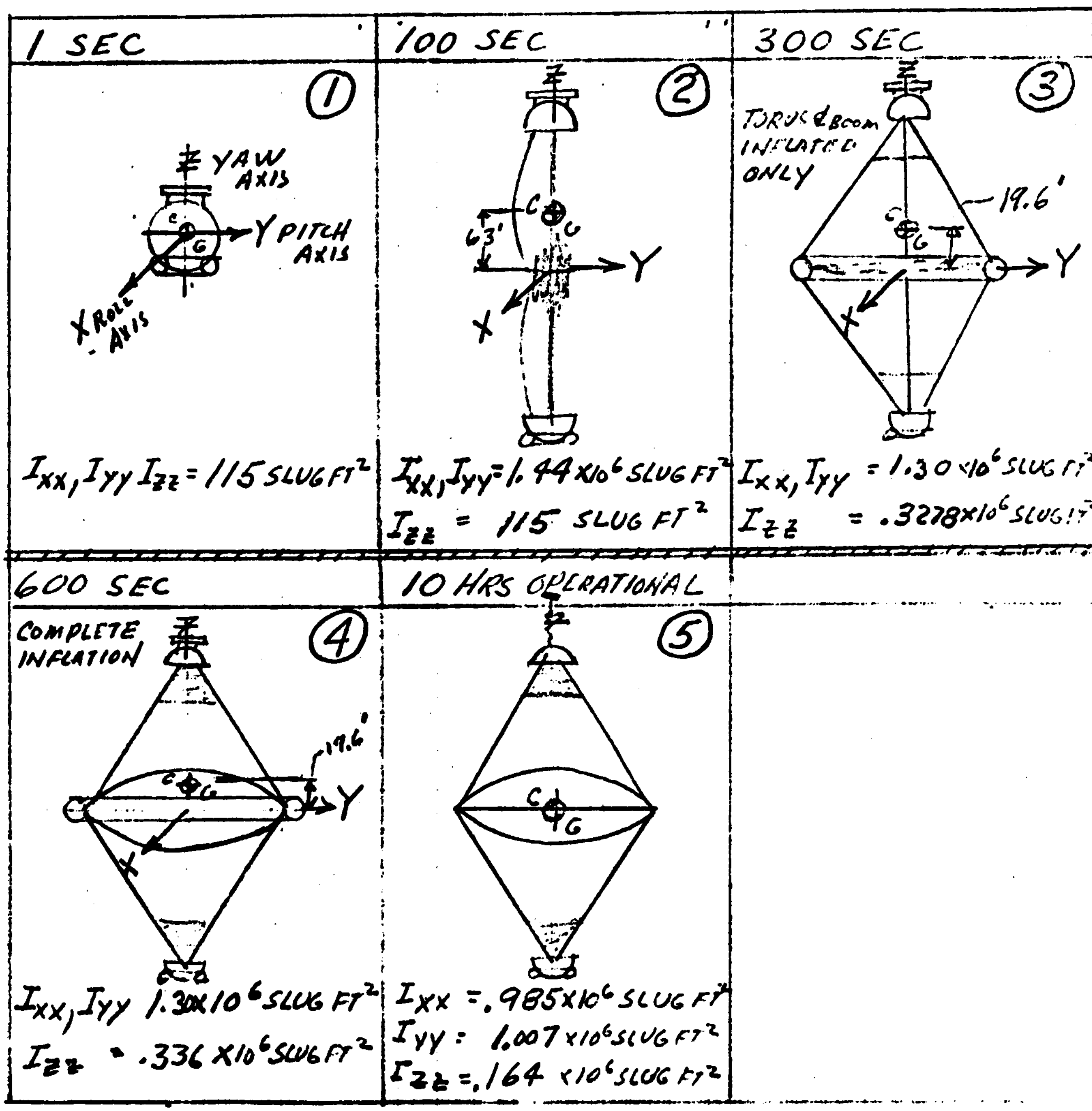
Figure 3 represents the satellite with torus and booms fully inflated, just prior to lens inflation

Figure 4 shows the satellite fully inflated and rigidized but before damper deployment

Figure 5 shows the operational stage of the satellite, i.e 10 hrs after separation, with the damper fully deployed, and the lens and torus fully photolyzed, and the yaw control masses properly oriented relative to the direction of the sails.

Calculations of moments of inertia of the satellite at these stages are given in pages 9 through 11, and summary table and graphical representation of moments of inertia and ratio I_{x-x}/I_{z-z} are given in page 12..

SYMMETRICAL CONFIGURATION



Moments of Inertia About Centroidal Axes During Various Stages of Deployment.

a. 1 sec.

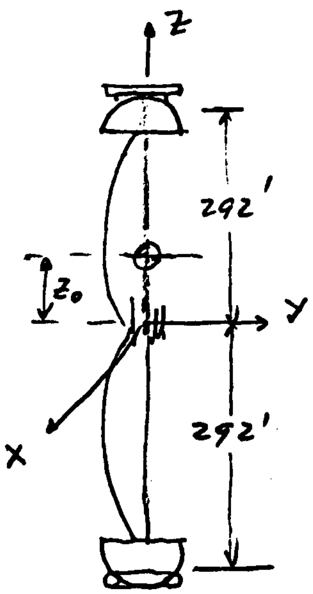
Total weight = 1484 lb.

Assume that the satellite weight is evenly distributed within a sphere of 2.5 Ft radius

Then $I_x = I_y = I_z = \frac{2}{5} W R^2 = \frac{2}{5}(1484)(2.5)^2 = 3710$ FT²-LB

$= 115$ SLUG-FT².

b. 100 sec.



WEIGHTS:

UPPER SIDE	LOWER SIDE	CENTER.	OTHERS.
184	184	812	150
105	8 (GAS)		22.
9 (GAS)			
298	192	812	172

(Separation system 10# off).

Centroid

$$z_0(298+192+812+172) = 106 \times 292 = 30952$$

$$z_0 = 30952/1474 = 21.0 \text{ Ft.}$$

Moments of inertia:

$I_{z\text{-}z}$ = (about the same as in 1 sec) 115 slug-FT²

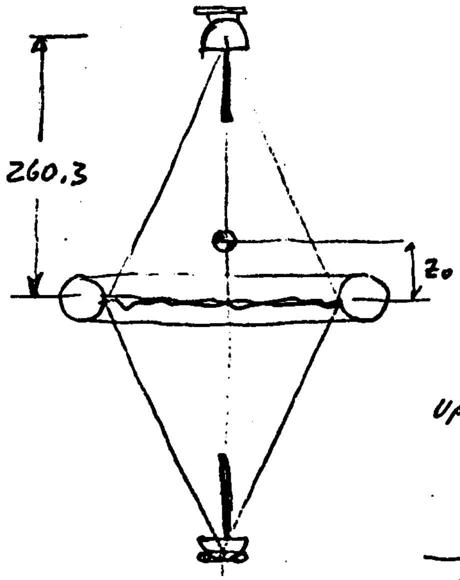
$$I_{x\text{-}y} = I_{y\text{-}y} = 298 \times (292-21)^2 + 192(292+21)^2 + 812 \times 21^2 + \frac{1}{3}(75)(292-21)^2 + \frac{1}{3}(75)(292+21)^2 + 11(292-58.8-21)^2 + 11(292+21-58.8)^2$$

$$= 323 \times 271^2 + 217 \times 313^2 + 812 \times 21^2 + 11 \times 212.2^2 + 11 \times 254.2^2$$

$$= 23{,}721{,}443 + 21{,}259{,}273 + 358092 + 495319 + 710{,}794 =$$

$46{,}544{,}921 \text{ lb-ft}^2 = 1.4455 \times 10^6 \text{ slug-ft}^2$

c. 300 sec.



(From Reference 2, page 149)

COMPONENT	$I_{(z-z)}$ (POLAR) LB-FT²	I_{x-x}, I_{y-y} LB-FT²
RIM	1,798,696	899,348
LENS (*)	5,187,496	3,089,019
TORUS	2,214,352	1,108,070
(*) Photolyzable film on		

Weights:

Upper side	Lower Side	Center	Others
184	184	812	150
105		8 (Gas)	22
9 (Gas)			
298 lb	184 lb	820 lb	172 lb.

Centroid:

$1474\, z_0 = (298-184) \cdot 260.3 = 29674.2 \; ; \; z_0 = 20.1 \text{ Ft}$

Except for the sail (and yaw control) the moments of inertia I_x & I_y coincide. Besides, the orientation of the satellite is uncertain so these two moments of inertia can be considered equal at this stage of deployment.

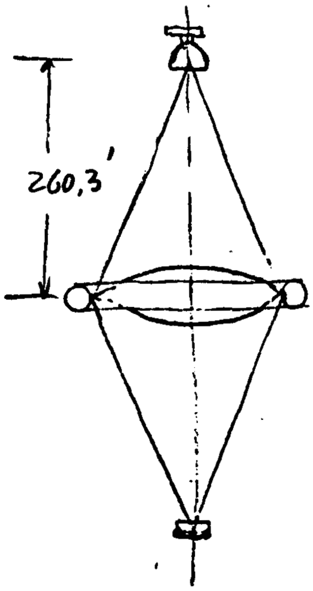
COMPONENT	WEIGHT	DISTANCE FROM CENTROID	MOMENTS OF INERTIA: I_x I_y (LB-FT²)	I_z (LB-FT²)
YAW CONTROL	40	20.1	0 to 732,580	716 420.
LENS	552	20.1	2,695,400	4,944,768
RIM	100.4	20.1	939,910	1,798,696
TORUS	117	20.1	1,155,340	2 214 352
UPPER MASS	298	240.2	17,193,408	0
LOWER MASS	184	280.4	14,466,845	0
UPPER SAIL	11	184.3	380,292	3924
LOWER SAIL	11	224.5	561,065	3924
UPPER TETRAND	75	110.0	1,554,755	447561
LOWER TETRAND	75	150.2	2,339,255	447561
Σ →			41,284,430 to 42,018,850	10,557,206

$I_{x-x} = I_{y-y} = (1.282 \text{ to } 1.305) \times 10^6$ slug-FT^2

$I_{z-z} = 0.3278 \times 10^6$ slug-ft^2

$I_{x-x}/I_{z-z} = 3.911$ to 3.981.

d. 600 sec. (Complete inflation).



Weights:

Upper Side	Lower Side	Center	Others
184	184	812	150
105		17 (Gas)	22
289 lb	184 lb	829 lb	172 lb.

Centroid:

$1474\, z_0 = (289-184) \times 260.3$

$z_0 = 18.5$ Ft.

For moments I_x & I_y same as before (300 sec).

Component	Weight	Distance from Centroid	Moments of Inertia (LB-FT²)(*) I_{x-x} or I_{y-y}	I_{z-z}
Yaw Control	40	18.5	0 to 730,110	716,420.
Lens	552	18.5	3,277,941	5,187,496.
Rim	100.4	18.5	923,710	1,798,696.
Torus	117	18.5	1,148,113	2,214,352
Upper Mass	289	241.8	16,897,032	0
Lower Mass	184	278.8	14,302,217	0
Upper Sail	11	185.9	386,807	3,924
Lower Sail	11	222.9	553,188	3,924
Upper Tetrap.	75	111.6	1,581,347	447,561
Lower Tetrapod	75	148.6	2,303,402	447,561
Σ →			41,373,757 or 42,103,367	10,819,934.

(*) The effect on I_x, I_z of the gas in the lens & torus was neglected.

$I_{x-x} = I_{y-y} = (1.285 \text{ to } 1.308) \times 10^6$ slug-ft^2

$I_{z-z} = 0.3360 \times 10^6$ slug-ft^2

$I_{x-x} / I_{z-z} = 3.824$ to 3.893.

Summary of Moments of Inertia During the Various Stages of Deployment and Graphical Representation of results.

ITEM	UNIT	SATELLITE STAGE DURING DEPLOYMENT				
		1 sec	100 sec.	300 sec	600 sec	10-Hours (OPERATIONAL)
WEIGHT	LB	1484	1474	1474	1474	880
I_{x-x} (ROLL)	SLUG-FT^2	115	1.4455×10^6	$(1.282-1.305) \times 10^6$	$(1.285-1.308) \times 10^6$	0.985×10^6
I_{y-y} (PITCH)	"	115	1.4455×10^6	"	"	1.007×10^6
I_{z-z} (YAW)	"	115	115	0.3278×10^6	0.3360×10^6	0.164×10^6
I_{x-x} / I_{z-z}	—	1.00	1.257×10^4	3.911–3.981	3.824–3.893	6.00
I_{y-y} / I_{x-x}	—	1.00	1.00	0.982–1.018	0.982–1.018	1.022

SYMMETRICAL LENSAT.

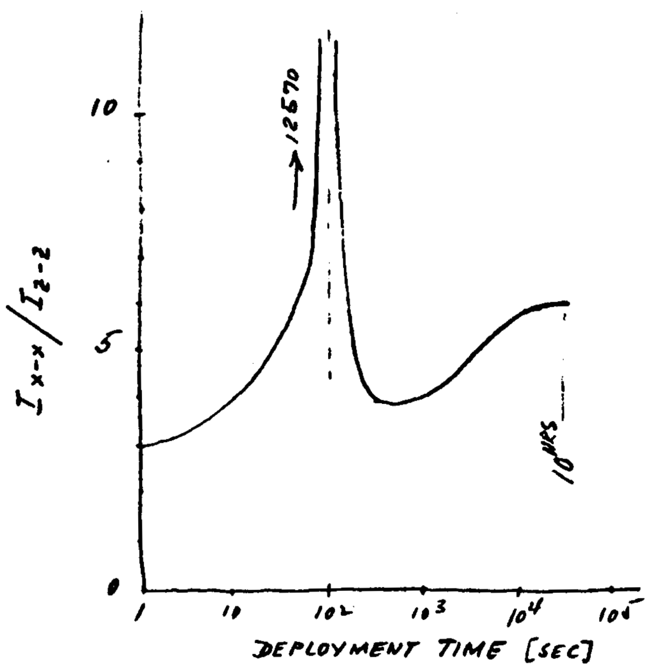


FIG. 1: RATIO I_{x-x}/I_{z-z} DURING DEPLOYMENT

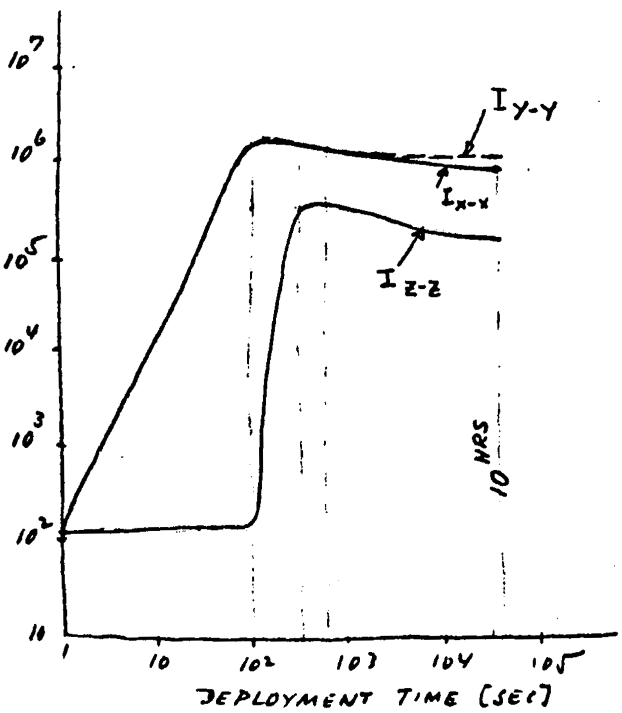


FIG. 2: VALUES OF I_{x-x}, I_{y-y}, I_{z-z} DURING DEPLOYMENT.

J. D. Marketos
D/456.

GOODYEAR AEROSPACE CORPORATION

ENGINEERING MEMORANDUM REPORT

DECEMBER 4, 1964
SM – 8828

Subject: Asymmetric Lensat Configuration with Ames Damper. Tetrapod size for $I_{x-x}/I_{z-z}=6.0$ (operational). Moments of Inertia about Principal Axes During Various Stages of Deployment.

REFERENCES

1. Feasibility Study and Preliminary Design of Gravity-Gradient-Stabilized Lenticular Test Satellite. Interim Technical Report, Contract NAS-1-31-14, GER-11502, June 1964.

2. Steel Construction. Manual of the American Institute of Steel Construction, Fifth Edition 1947, New York, N.Y.

VDM

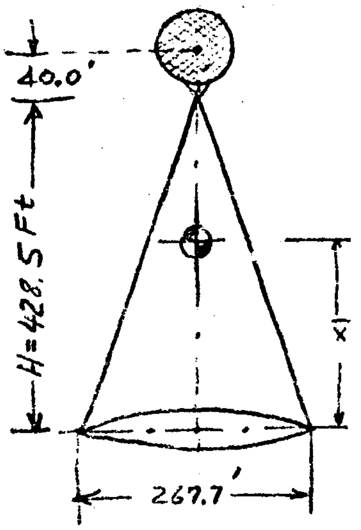
DEC. 2/64

LENTICULAR SATELLITE

ASYMMETRICAL CONFIGURATION.

Preliminary calculations showed that the hight of the tetrapod (for $I_{(ROLL)}/I_{(YAW)} = 6.0$) must be 428.5 Ft.

1. CENTROID LOCATION



① COMPONENT	② WEIGHT	③ DISTANCE FROM LENS CG	④ ② × ③
LENS	199.0	0	0
RIM	100.4	0	0
BOOMS (*)	75.0	214.25	16 068.8
DRIVE SYSTEM PLUS UPPER MAS	287.0	428.5	122 979.5
RODS & MAS	126.0	428.5	53 991.0
SPHERE	19.0	468.5	8901.5
BALANCED SAIL	22.0	$\bar{x}$	$22.0\,\bar{x}$
$\Sigma \rightarrow$	828.4		

$$806.4\,\bar{x} = 201{,}940.8, \quad \bar{x} = 250.42 \text{ Ft}$$

2. SIZE OF BALANCING SPHERE, (By Proportioning from Westinghouse's configuration — H=100', +30' for sphere center; Sph. DIA = 37.5 Ft., satellite centroid 141 Ft from Lens-Rim centroid)

$$A\,(141) = \pi (37.5)^2 (430 - 141) \qquad (A: \text{Lens effective area})$$
$$A\,(250.42) = \pi D^2 (468.5 - 250.42)$$

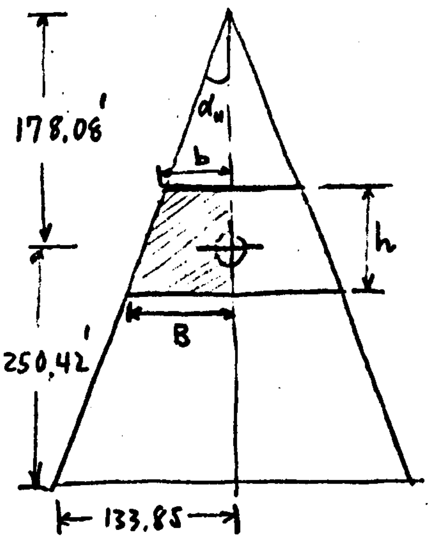
Dividing these two equations and solving for D yields,

$$D = 37.5 \sqrt{\frac{250.42 \times 289}{141 \times 218.08}} = 37.5\sqrt{2.3535} \cong 57.6 \text{ Ft.}$$

Assume $\frac{1}{4}$ mil MYLAR. $WT = \pi (57.6)^2 (144) \left(\frac{1}{4}\right) 10^{-3} (0.05) = 18.76 \text{ lb} \cong 19$ (CHECK)

(*) The weight of the booms was taken arbitrarily equal to 75 lb for all four booms (only wire material).

3. SIZE & LOCATION OF BALANCED SAIL.



Area : 8000 FT^2

$\tan\alpha_u = 0{,}31247$

$\alpha_u = 17^\circ\, 21'$

Boom length $= (133{,}85^2 + 428{,}5)^{1/2} = 448.92$ Ft

$\sin\alpha_u = 0.29816$

$\cos\alpha_u = 0.95451$

Equations :

$$\begin{cases} (b+B)\,h = 8000 \\ B = b + 0{,}31247\,h \\ B = \left(178{,}08 + \dfrac{h}{3}\,\dfrac{2b+B}{b+B}\right)(0{,}31247) \end{cases}$$

Elimination of b & B leads to

$$\left(\frac{h}{100}\right)^4 - 27{,}355\left(\frac{h}{100}\right) + 19.664 = 0.$$

Solving this equation yields :

$h = 72{,}90$ Ft.

Then, $b = 43.50$ Ft

$B = 66.25$ Ft.

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4. MOMENTS OF INERTIA ABOUT CENTROIDAL AXES OF VARIOUS COMPONENTS.

4A) SPHERE.

Total weight $W = 19$ #.

Weight per sq. foot of surface : w

Moment about any centroidal axis

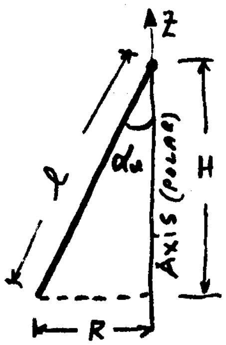
$$I = \frac{2}{3}(4\pi R^2 w)R^2 = \frac{2}{3}WR^2$$

For $R = 28.8$ FT, $W = 19$,

$$I = \frac{2}{3}(19)(28.8)^2 = 10506.\ FT^2\text{-}LB.$$

4b). TETRAPOD. (SET OF 4 BOOMS)

Total weight of 4 booms W (=75 lb).



Moment of one boom about axis shown in sketch.

$$I = \frac{1}{3}\left(\frac{W}{4}\right)(\ell \sin\alpha_u)^2 = \frac{WR^2}{12}$$

Polar moment of inertia of tetrapod

$$I_Z = 4I = \frac{WR^2}{3}$$

For $W = 75$, $R = 133.85$; $I_Z = 447,895$ LB-FT^2.

Moment of inertia of all four booms about <u>any</u> axis passing through the apex & normal to the polar axis : I'.

$$I' = 2I + \frac{1}{3}\left(\frac{W}{4}\right)(\ell\cos\alpha_u)^2(4) = \frac{WR^2}{6} + \frac{WH^2}{3}$$

Moment of inertia about <u>any</u> centroidal axis normal to the polar axis,

$$I_x = I_y = I' - W\left(\frac{H}{2}\right)^2 = \frac{WR^2}{6} + \frac{WH^2}{12} = \frac{W}{12}(R^2 + \ell^2)$$

Hence $I_x = I_Y = \frac{75}{12}(133.85^2 + 448.92^2) = 1,371,513$ LB-FT^2.

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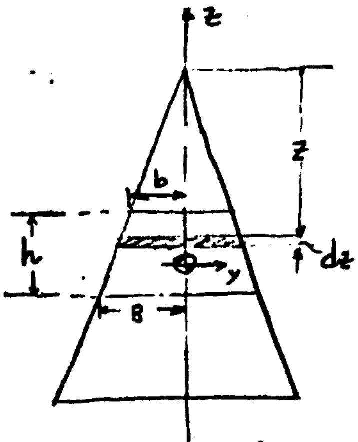
4c.) SAIL.



w = weight per unit area = $\frac{22}{8000} = 0.00275$ lb/ft^2.

(a) About z-axis

$$dI_z = \frac{1}{12}\,dz\,(2y)^3 w = \frac{2}{3}\,w\,y^3\,dz.$$

But $y = 0.31247\,z$

$$\therefore\; I_z = \frac{2}{3}(0.31247)^3\left(\frac{1}{4}\right)w\,\frac{B^4 - b^4}{(0.31247)^4}$$

$$= \frac{w}{6(0.31247)}\left[66.25^4 - 43.50^4\right]$$

$$= \frac{0.00275}{1.87482}(19{,}263{,}874 - 3{,}580{,}610) = 23{,}004 \text{ LB-FT}^2$$

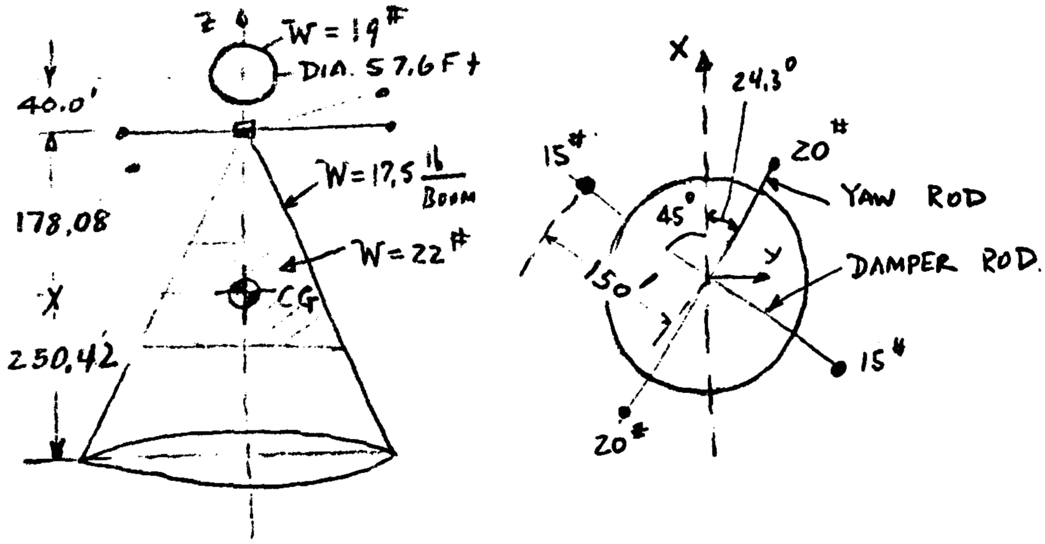
From Reference 2, Page 361,

$$I_Y = w\,\frac{h^3(4)(B^2 + 4Bb + b^2)}{36\,(2)\,(B+b)} = \frac{72.9^3 \times w}{18 \times 109.75}\left(66.25^2 + 4 \times 66.25 \times 43.50 + 43.50^2\right)$$

$$= 196.11\,(4389.1 + 11{,}527.5 + 1892.2)(0.00275) = 9604 \text{ lb-Ft}^2.$$

$$I_{polar} = I_z + I_y = I_x = \underline{23004 + 9604 = 32{,}608} \text{ lb-Ft}^2.$$

5. WEIGHTS & MOMENT OF INERTIA AT FINAL STAGE OF DEPLOYMENT



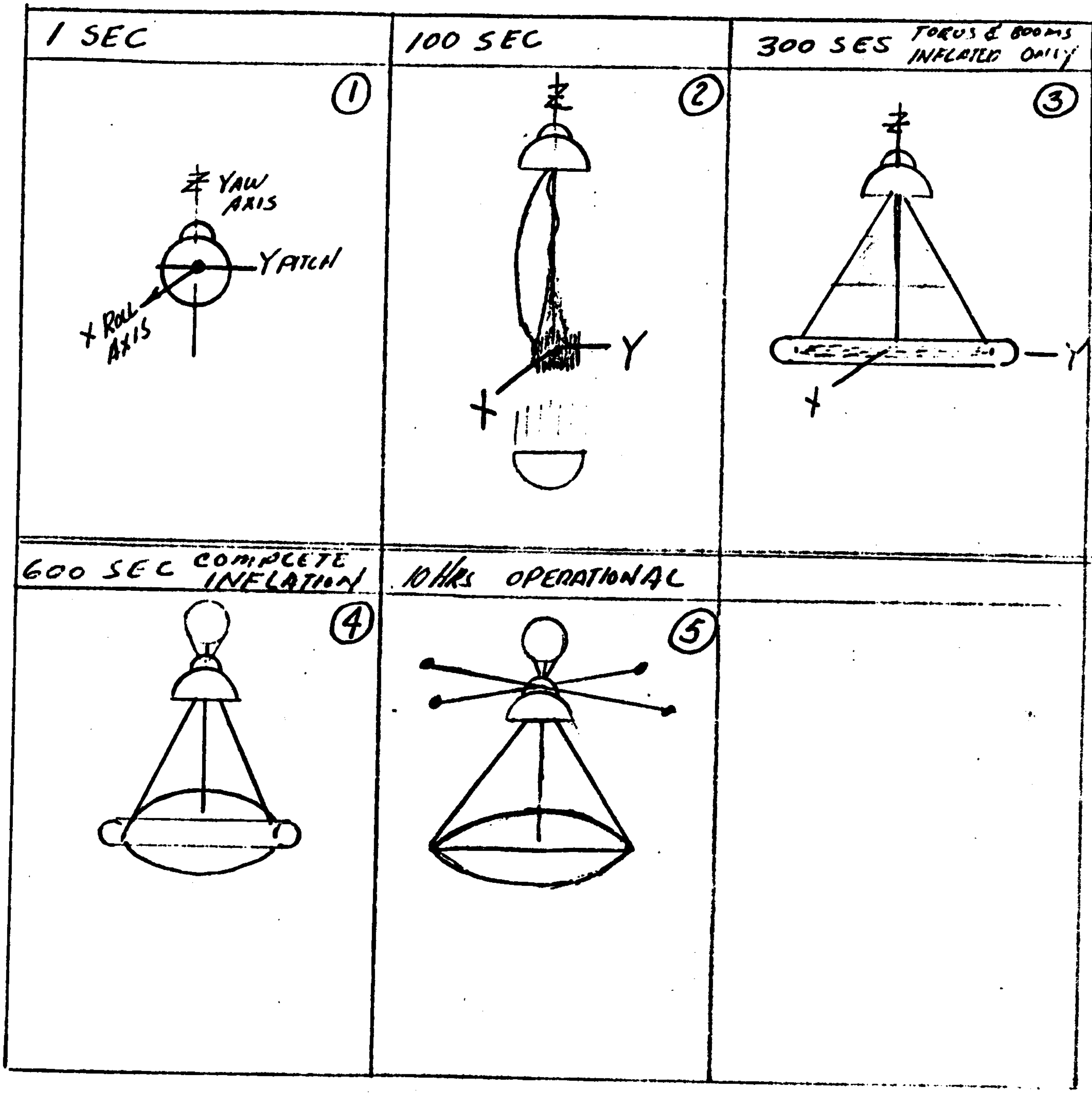
COMPONENT	WEIGHT [lb]	Distance of Component cg From System's Centroidal axis x	y	z	I_{x-x} (ROLL) LB-FT^2	I_{y-y} (PITCH) LB-FT^2	I_{z-z} (YAW) LB-FT^2
LENS	199.0	250.42	250.42	0	13,592,205	13,592,205	1,868,896
RIM	100.4	250.42	250.42	0	7,195,450	7,195,350	1,798,696
APEX MASS	287.0	178.08	178.08	0	9,101,485	9,101,485	0
YAW ROD	28.0	178.08	178.08	0	923510	11,062,387	210,000
YAW MASSES(2)	40.0	—	—	150	1420924	2,016,084	900,000
DAMPER ROD	28.0	178.08	178.08	0	927573	927573	210,000
DAMPER MASSES(2)	30.0	—	—	150	1288881	1288881	675,000
SAIL	22.0	0	0	0	32608	9604	23,004
SPHERE	19.0	218.08	218.08	0	914,871	914,871	10,506
BOOMS (4)	75.0	36.17	36.17	0	1469,633	1,469,633	447,895
Σ →	828A				36,867,140	37,578,073	6,143,997

$$\frac{I\,(\text{ROLL})}{I\,(\text{YAW})} = 6.000 \qquad \frac{I\,(\text{PITCH})}{I\,(\text{ROLL})} = 1.019.$$

11-23-64
HWB

6. VARIOUS STAGES OF DEPLOYMENT.

The sketches below show LENSAT in the process of deployment at the indicated times.



MOMENTS OF INERTIA ABOUT CENTROIDAL AXES (REFERENCE 1, PAGE 149)

COMPONENT	I_{z-z} (POLAR) FT^2-LB	I_{x-x}, I_{y-y} (IN-PLANE) FT^2-LB
RIM	1,798,696	899,348
LENS (*)	5,187,496	3,089,019
TORUS (*)	2,214,352	1,108,070

(*) WITH PHOTOLYZABLE FILM.

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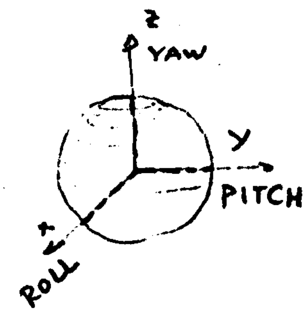
7. MOMENTS OF INERTIA ABOUT CENTROIDAL AXES DURING VARIOUS STAGES OF DEPLOYMENT.

a). 1 SEC.



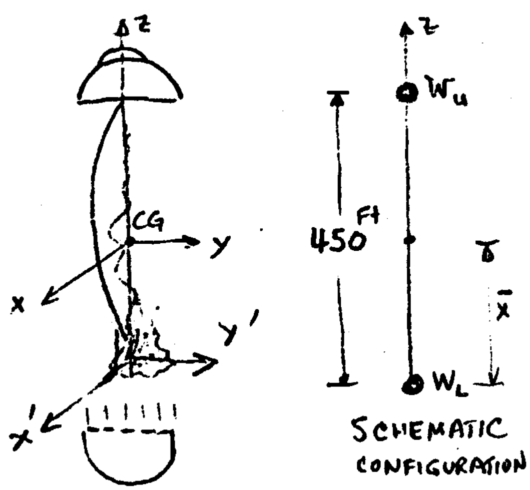
TOTAL WEIGHT = 1368 lb.

Assume that the satellite weight is evenly distributed within a sphere of 2,5 Ft radius.

Then $I_x = I_y = I_z \approx \frac{2}{5} W R^2 = \frac{2}{5}(1368) \times 6.25 = 3420$ LB-FT^2

$= 106.2$ SLUG-FT^2.

b). 100 SEC.



SCHEMATIC CONFIGURATION

Weights:

UPPER SIDE WT. 413 LB.
SPHERE 19 "
GAS 17 "
W_u = 449 LB.

LOWER SIDE.:
W_L = 772 LB.
BOOMS-SAIL = 75+22 = 97 LB.

CENTROID:

$(449+772+97)\bar{x} = 449 \times 450 + 97 \times 225$

$1318\,\bar{x} = 223{,}875$

$\bar{x} \approx 170.0$ Ft.

Moments of inertia:

I_{zz} = (about the same as in 1 sec) = 106.2 SLUG-FT^2

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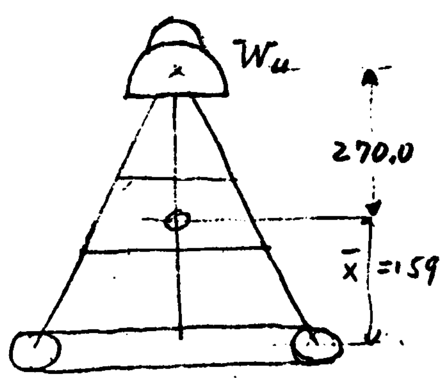
$I_{x-x} = I_{y-y}$

$$\begin{aligned}
\text{Upper mass} &= 449 \times (450-170)^2 &&= 35{,}201{,}600 \text{ LB-FT}^2 \\
\text{Lower mass} &= 772 \times 170^2 &&= 22\,310{,}800 \quad " \\
\text{Booms: } &\begin{cases} (75/12) \times 450^2 \\ 75(225-170)^2 \end{cases} &&\begin{aligned} &= 1{,}265{,}625 \quad " \\ &= \underline{\quad 226{,}875 \quad "} \end{aligned} \\
& && \quad 59{,}004{,}900
\end{aligned}$$

$$I_{x-x} = I_{y-y} = 59{,}004{,}900 \text{ LB-FT}^2 = 1{,}832{,}450 \text{ SLUG-FT}^2$$

c). 300 sec.



Weights.

$W_u = 413 + 19 = 432$ LB

SAIL $= 22$ LB

BOOMS $= 75$ LB

TORUS $= 117$ LB

Lens $= 199 + 353 = 552$ LB.

Rim $= 103$ LB

C.G LOCATION: $1301(\bar{x}) = 250(22.0) + 75(214.5) + 432 \times 428.5$

$$\bar{x} = \frac{1}{1301}(5500 + 16088 + 185{,}112) \cong 159 \text{ Ft.}$$

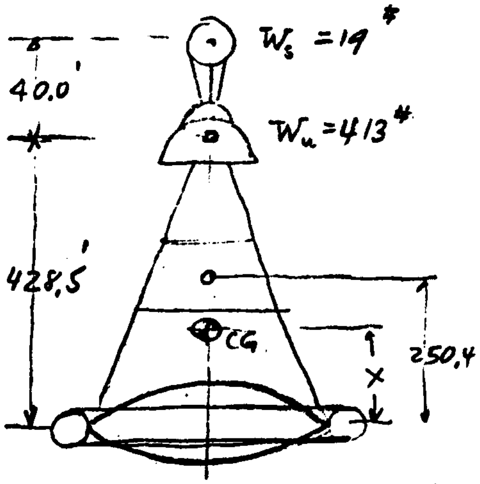
COMPONENT	WEIGHT [LB]	DISTANCE FROM SYSTEM CENTROID [FT]	MOMENTS OF INERTIA I_x	I_y	I_z
W_u	432	270	31,492,800	EXCEPT FOR THE SAIL THESE MOMENTS OF INERTIA COINCIDE WITH I_x; BESIDES THE ORIENTATION OF THE SATELLITE IS UNCERTAIN, SO I_Y AND I_X CAN BE CONSIDERED EQUAL @ THIS PHASE	$\cong 0$
BOOMS	75	55.5	1,602,532		447 895
SAIL	22	91.0	203,288		23,004
TORUS	117	159	4,065,947		2,214,352
RIM	103	159	3,503,291		1,798,696
LENS	552	159	16,427,496		4,944,768
$\Sigma \rightarrow$	1301		57,295,354		9,428,715

$I_x / I_z = 6.077$ ($I_{x-x} = 1{,}779{,}359$ SLUG-FT^2; $I_{z-z} = 292{,}817$ SLUG-FT^2)

JDM

DEC, 3/64.

d). 600 Seconds, (Complete Inflation).



CG. LOCATION:

$$1301\,\bar{x} = 250(22) + 75(214.3) + 413 \times 428.5 + 19 \times 468.5 = 207,444$$

$$\bar{x} = 159.4$$

COMPONENT	WEIGHT [LB]	DISTANCE FROM SYSTEM CENTROID [FT]	MOMENTS OF INERTIA $I_x = I_y$ (See case 3, 300sec)	I_z (Polar)
W_s	19	309.1	1,825,819	10506
W_u	413	269.1	29,907,312	—
BOOMS	75	54.9	1,597,563	447895
SAIL	22	91.0	203,288	23004
TORUS	117	159.4	4,080,484	2 214,352
RIM	103	159.4	3,516,409	1,798,696
LENS	552	159.4	17,114,434	5,187,496
Σ —	1301		58,245,309	9,681,949

$$\frac{I_x}{I_z} = 6.016 \qquad I_{x-x} = 1,808,860 \text{ SLUG FT}^2;\ I_{z-z} = 300,682 \text{ SLUG-FT}^2$$

e). 10 HRS — OPERATIONAL. (Torus-Lens-Booms Photolyzed; Yaw and damper rod deployed).

AS SHOWN IN THE ANALYSIS (Page 5).

$I_{x-x} = 36\,867,140$ LB-FT^2 = 1,144,942 SLUG-FT^2

$I_{Y-Y} = 37,578,073$ " = 1,167,021 "

$I_{Z-Z} = 6,143,997$ " = 190,807 "

$$\frac{I_{x-x}}{I_{z-z}} = 6.000 \;;\quad \frac{I_{Y-Y}}{I_{x-x}} = 1.019$$

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8. SUMMARY OF MOMENTS OF INERTIA DURING THE VARIOUS STAGES OF DEPLOYMENT, AND GRAPHICAL REPRESENTATION OF RESULTS.

ITEM	UNITS	SATELLITE PHASES DURING DEPLOYMENT				
		1 SEC	100 sec	300 sec	600 sec	10-HRS OPERATIONAL
WEIGHT	LB	1368	1318	1301	1301	831
I_{x-x} (ROLL)	SLUG-FT²	106.2	1,832,450	1,779,359	1,808,860	1,144,942
I_{y-y} (PITCH)	"	106.2	1,832,450	1,779,359	1,808,860	1,167,021
I_{z-z} (YAW)	"	106.2	106.2	292,817	300,682	190,807
I_{x-x}/I_{z-z}	—	1.00	17,255.00	6.077	6.016	6.000
I_{y-y}/I_{x-x}	—	1.00	1.00	1.00	1.000	1.019

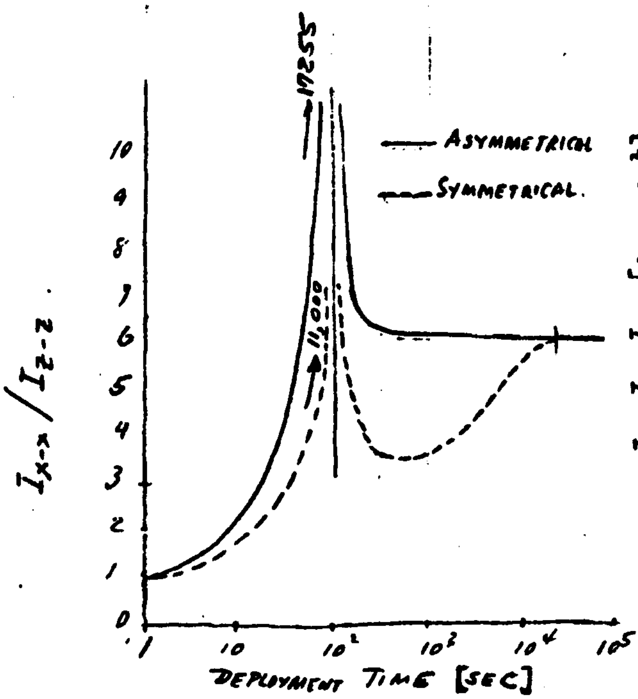

FIG. : RATIO $\frac{I_{x-x}}{I_{z-z}}$ DURING DEPLOYMENT

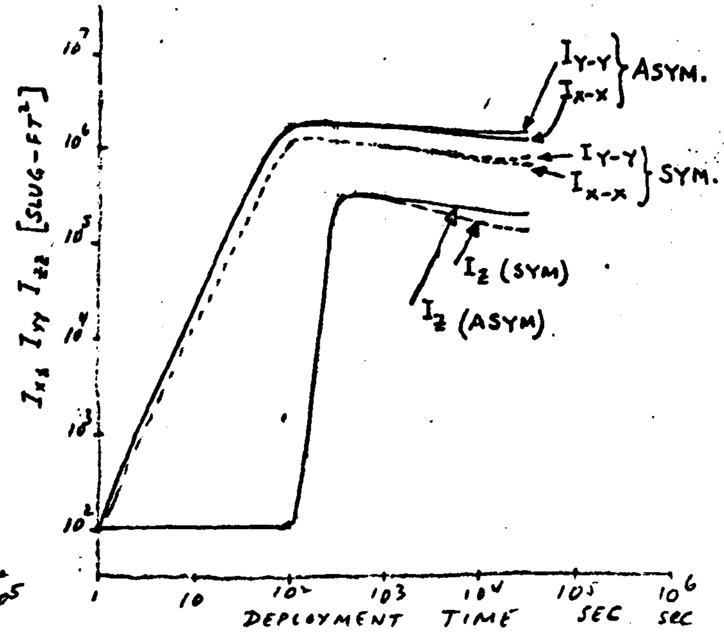

FIG. : VALUES OF I_{xx}, I_{yy} I_{zz} DURING DEPLOYMENT.

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D/456.